Note Title 2/10/2010

STT 200 3pm 2-10-10

Ch18, PREVIEW CH 19, IN CLASS ASSIGNMENT.

BERNOULLI TRIALS: Y $P(Y) = p$; N $P(N) = q$; n INDEP TRIALS —

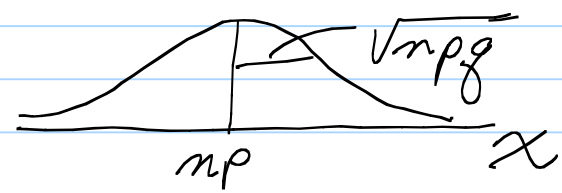
BINOMIAL: X = # Y IN n TRIALS.

eg X = # HEADS (Y) IN $n = 100$ TOSSES OF COIN.

(PERSI DIACONIS) DISCRETE PR $P(X=x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$

w/ $0! \overset{DEF}{=} 1$, eg $3! = 3(2)(1)$ $\qquad x = 0, 1, \ldots, n$

NOTICE $n \to \infty$, $p \not\approx 0$, $p \not\approx 1$, ie $\left.\begin{array}{l} np \geq 10 \\ nq \geq 10 \end{array}\right\} \Rightarrow \sim$



WE ARE IN MANY INSTANCES INTERESTED IN $\hat{p} = X/n$

SO $\hat{p}$ = SAMPLE PROPORTION ANSWERING "Y"

Eg DRAW A SAMPLE OF $n = 62$ PARTS.

FINDING $X = 14$ ARE DEFECTIVE

p = POPULATION FRACTION OF DEFECTIVE PARTS

SO $\hat{p}$ (OUR EST OF p) IS $\hat{p} = 14/62$

EXAMPLE OF USE OF $\hat{p}$ MIGHT BE

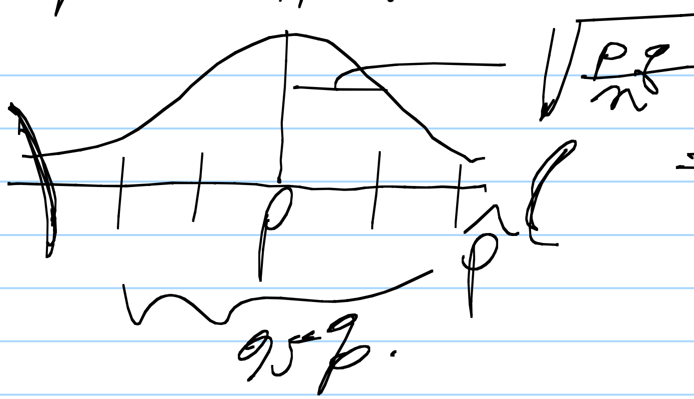
"SAMPLE $n = 62$ "PARTS" - TEST THESE

$$\hat{p} = \frac{X}{n} = \frac{\text{\# OF 62 FOUND DEFECTIVE}}{62}$$

REJECT SHIPMENT IF $\hat{p} > 0.1$ (JUST EXAMPLE)

ANOTHER IMPROVEMENT OF NORMAL APPROX.

$\hat{p} \sim$



$p \neq 0$

$p \neq 1$

$$\Rightarrow P\left(|\hat{p}-p| \le 2\sqrt{\frac{pq}{n}}\right)^{*} \approx .95$$

STATEMENT* ABOVE WOULD BE USEFUL IF $\sqrt{pq/n}$ IS KNOWN. WONDERFULLY, IT IS ALSO TRUE THAT

$$P\left(|\hat{p}-p| \le 2\sqrt{\hat{p}\hat{q}/n}\right) \text{ ALSO } \approx .95$$

TO SEE HOW THIS IS USED:

IF SAMPLE $n = 100$ PARTS. SO $\hat{p} = \frac{X}{n} = 0.42$

FINDING $X = 42$ DEFECTIVE

LOOK AT INTERVAL $\hat{p} \pm 2\sqrt{\hat{p}\hat{q}/n}$

$$.42 \pm 2\sqrt{\frac{.42 \cdot .58}{100}} \approx 0.1$$

NOT REAL ACCURATE

p IN RANGE $.42 - .1 \,,\, .42 + .1$ $.098$

$(.32 \,,\, .52)$.

EITHER THE INTERVAL " ENCLOSES p OR NOT.

FAT GUESS AT p: $\hat{p} \pm 2\sqrt{\hat{p}\hat{q}/n}$.

$$P\left(p \text{ "IN" } \boxed{\hat{p} \pm 2\sqrt{\hat{p}\hat{q}/n}}\right) \sim .95$$

$\hat{p}$ = EST OF p

EST OF SD OF $\hat{p}$.

LIKEWISE $P\left(p \text{ "IN" } \hat{p} \pm 1\sqrt{\frac{\hat{p}\hat{q}}{n}}\right) = .68$

EXAMPLE OF USE. X = # OF STUDENTS WITH EVEN STUDENT #. IN SAMPLE OF n

$n = 23 \quad X = 12$

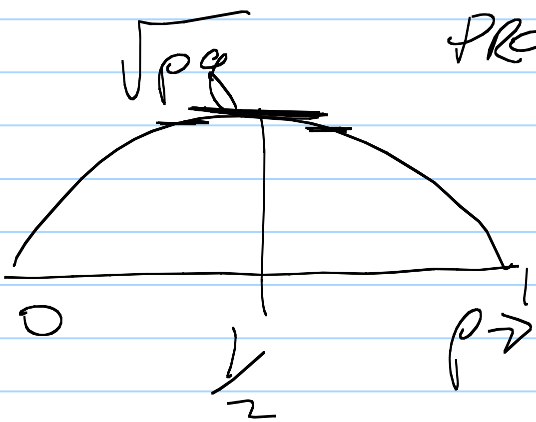
FAT GUESS $\quad \frac{12}{23} \pm 2\sqrt{\frac{12/23 \;\; 11/23}{23}}$

$\hat{p} \pm 2\,\hat{\sigma}_{\hat{p}}$

$\sigma_{\hat{p}} = \sqrt{pq/n}$

CLAIM: MY SAMPLE OF $n=23$

PRODUCED $\hat{p} \pm 2\,\hat{\sigma}_{\hat{p}}$ $\qquad \sqrt{\frac{12}{23}\frac{11}{23}} \sim \frac{1}{2}$



$$\hat{p} \pm 2\sqrt{\hat{p}\hat{q}/n}$$

$$\frac{12}{23} \pm 2\sqrt{(12/23 \cdot 11/23)/23}$$

$$\frac{12}{23} \pm 1/\sqrt{23} = [.313\ ,\ .730]$$

95% CONFIDENCE INTERVAL FOR p

FOR ASSIGNMENT USE 68% C.I.

$$\hat{p} \pm 1\sqrt{\hat{p}\hat{q}/n}$$

1. EACH OF YOU IS USING YOUR OWN BLOCK OF Y + N (PUT YOUR NAME ABOVE YOUR BLOCK)
2. FORM 68% CI $\hat{p} \pm 1\sqrt{\hat{p}\hat{q}/n}$
3. I WILL LATER TELL YOU WHAT YOUR p (TRUE p) IS. FOR EACH BLOCK.
4. YOU WILL THEN SAY (ON THE SHEET BY YOUR NAME) WHETHER YOUR (TRUE) p IS ENCLOSED BY YOUR 68% CI.
5. IF ALL GOES WELL AROUND 68% OF STUDENTS IN THE CLASS WILL FIND THAT THEIR OWN 68% CI COVERS THEIR TRUE p.

.5

.6

.4

$n = 100$

100

100

11 COVER

6 NOT

$\approx 68\%$ OF 17

CI ATTEMPTS

.5] 30

.6] 30

.4] 30

.5] $n = 80$

.6] $n = 60$

.4] $n = 60$