STT 200 5:30pm 3-15-10

Note Title 3/15/2010

CH 23/24. CI & TESTS ABOUT A POPULATION MEAN μ (mu).

RECOLLECT "LARGE n" (np & $nq \geq 10$, GUESS AT THIS USING $n\hat{p}$, $n\hat{q}$).

z-CI $\hat{p} \pm z \dfrac{\sqrt{\hat{p}\hat{q}}}{\sqrt{n}}$ — EST^d SD OF $\hat{p}$ — (EST^d) STANDARD ERROR OF $\hat{p}$!

RECALL:

$$\sigma_{\hat{p}} = \frac{\sqrt{pq}}{\sqrt{n}}$$

CH 23. WANT CI NOT LIMITED TO PROPORTIONS.

BUT APPLYING TO ANY POPULATION MEAN SCORE.

eg X = BODY TEMP μ = (THINK) 98.6 °F.

"LARGE n SAMPLE" WE NEED TO EST POP SD

$$\sigma_{pop} = \sqrt{\frac{(x_1-\mu)^2+(x_2-\mu)^2+\cdots+(x_N-\mu)^2}{N}}$$

N = POP SIZE

= ROOT MEAN SQUARED DEVIATION FROM AVG.

ABSENT KNOWLEDGE OF σ_{pop} WE ESTIMATE IT THUS:

$$s = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\cdots+(x_n-\bar{x})^2}{\cancel{n}\ (n-1)}}$$

(i.e. s) sample std deviation

THEN OUR EST OF $\sigma_{\bar{x}}$ IS $\frac{\alpha}{\sqrt{n}}$

BECAUSE

$$\sigma_{\bar{x}} = \sigma_{pop}/\sqrt{n}$$

$$\mathrm{Var}\,\bar{X} = \mathrm{Var}\left(\frac{X_1 + \cdots + X_n}{n}\right) = \frac{1}{n^2}\mathrm{Var}(X_1 + \cdots + X_n) \overset{\text{INDEP}}{=} \frac{\mathrm{Var}\,X_1 + \cdots + \mathrm{Var}\,X_n}{n^2}$$

$$= \frac{n\,\mathrm{Var}\,X_1}{n^2} = \frac{n\,\sigma_{pop}^2}{n^2} = \sigma_{pop}^2/n$$

SO TAKING ROOT

$$\sigma_{\bar{x}} = \sigma_{pop}/\sqrt{n}$$

SO WHY NOT EST IT BY $\frac{\alpha}{\sqrt{n}}$

WE "SUSPECT" THAT 95% z-BASED CI FOR μ

IS $\bar{x} \pm z \; \frac{s}{\sqrt{n}}$

EST OF μ — 1.96 — EST OF $\sigma_{\bar{x}}$

EXAMPLE 1.

APPLY TO CONTEXT OF x = LAST DIGIT OF STUDENT #

POP

0 1 2 .. 9

KNOW

$\mu = 4.5$

$\sigma_{Pop} = 2.87$

I C. WITH REPL SAMPLE $n = 25$

3, 8, . . . , 2, 7, 7

$\bar{x} = (3+8+\cdots+7+7)/10 = 5.13$

$s = \sqrt{\frac{(3-5.13)^2+\cdots+(7-5.13)^2}{\cancel{25}\ (25-1)}}$ (SAY NOT)

CONFIRM:

$\sigma_{Pop} = \sqrt{\frac{(0-4.5)^2+\cdots+(9-4.5)^2}{10}}$

~ 2.87

SAY $\sigma = 2.88$ (SAY) NOT

$\Rightarrow$ 95% z-BASED CI FOR μ: $5.13 \pm 1.96 \frac{2.88}{\sqrt{25}}$

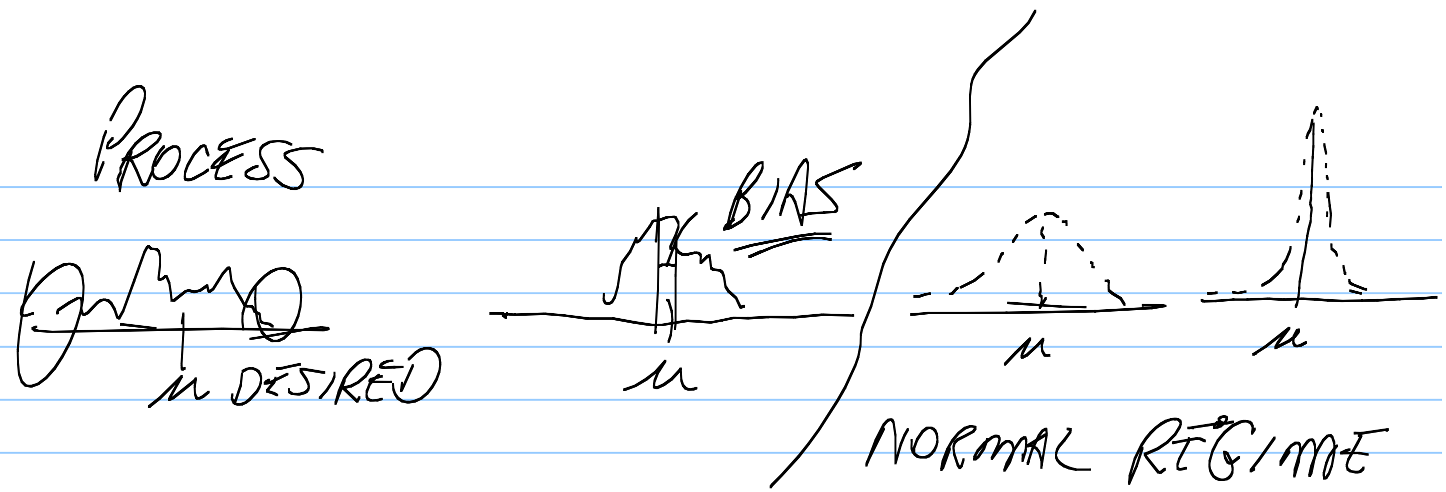
$\bar{x} \pm z \, \sigma/\sqrt{n}$ SO $\nearrow$

$z \sim 1.96$

HOW TO MODIFY IF POPULATION SIZE IS $N = 700$ AND $n = 25$ IS WITHOUT REPLACEMENT?

$$\bar{x} \pm z \, \sigma/\sqrt{n} \sqrt{\frac{N-n}{N-1}}$$ $\leftarrow$ FPC.

REMARKABLY WE CAN HANDLE ALL $n > 1$ IF THE POPULATION IS NORMAL.



GOSSETT - IF POPULATION IS NORMAL (OR NEARLY SO)

DIST OF $\frac{\bar{X}-\mu}{s/\sqrt{n}}$ DOES NOT DEPEND UPON μ OR σ $\Rightarrow$ CAN TABULATE THE DISTRIBUTION IF POP^N IS NORMAL

FORM t-BASED CI FOR μ

VALID FOR EVERY $n > 1$ (IF POP IS NORMAL) t-BASED CI FOR μ:

$$\bar{x} \pm z\ t\ s/\sqrt{n}$$

example: suppose #2, $n = 7$ FROM NORMAL Pop^{N}.

Let's suppose $\bar{x} = 97.2$ (NOT), $s = 14.81$ (NOT σ)

FOR 95% CI; Deg Freedom (ACTUALLY $(n-1)$ IN t-BASED CI)

$7-1=6$	2.365
∞	1.96
Confidence level	95%

95% t-BASED CI is $\bar{x} \pm t\ s/\sqrt{n}$

$n=7$

(DF $= 7-1 = 6$) $97.2 \pm 2.365 \dfrac{14.81}{\sqrt{7}}$

CLAIM: $P\left(\mu \text{ IN } \boxed{\bar{x}_{\ldots} \pm 2.365_{\ldots} \frac{s_{\ldots}}{\sqrt{7}} \ldots}\right) = .95$

FOR $n=7$

ALSO $P\left(\mu \text{ IN } \boxed{\bar{x}_{\ldots} \pm 2.093_{\ldots} \frac{s_{\ldots}}{\sqrt{20}}}\right) = .95$

$t_{DF = 20-1 = 19}$

DF	
6	2.365
19	2.093
∞	1.96

95%

INCR

"ONLY" PRICE PAID BY USING s TO EST σ_{pop}.

PRICE OF SMALL n IS IN THE $s/\sqrt{n}$

WHAT ABOUT TESTS?

$H_0: \mu = 98.6$

$H_A: \mu > 98.6$

t SCORE

DF = 50 − 1

DATA FROM NORMAL POPULATION

FIND ($n = 50$) $\bar{x} = 99.4$

$s = 0.83$

TEST STAT: $\dfrac{\bar{x}-\mu_0}{s/\sqrt{n}} = \dfrac{99.4-98.6}{0.83/\sqrt{50}}$

SUPPOSE IT WORKS OUT TO

TEST STAT = 2.3 NOT

DF				
~~49~~ 50	1.299	1.676	2.009	2.403
CONF			95%	98%

2.3

t_{49} DF

2.5%

.1

2.009 2.403