Note Title 3/15/2010

WHEN OBJECT IS TO ESTIMATE POPULATION MEAN μ

HAVE DATA IN FORM OF INDEP SAMPLES (THINK: WITH-REPLACEMENT).

$$\bar{x} = \frac{x_1 + \cdots + x_n}{n} \quad \text{ESTIMATOR OF } \mu.$$

WHAT IS STANDARD DEVIATION OF $\bar{x}$?

$$\text{KNOW } \operatorname{Var} \bar{X} = \operatorname{Var}\left(\frac{X_1 + \cdots + X_n}{n}\right) = \frac{1}{n^2} \operatorname{Var}(X_1 + \cdots + X_n)$$

$$\overset{\text{INDEP}}{=} \frac{1}{n^2}\left(\operatorname{Var} X_1 + \cdots + \operatorname{Var} X_n\right) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

SO $\sigma_{\bar{X}} = \sigma_{pop}/\sqrt{n}$

$\sigma = \sqrt{\dfrac{(x_1-\mu)^2 + \cdots + (x_N-\mu)^2}{N}}$

GUESS: THAT APPROPRIATE CI FOR μ

$\bar{X} \pm z$ EST OF $\sigma/\sqrt{n}$

EST σ MOST NATURALLY BY $s = \sqrt{\dfrac{(x_1-\bar{x})^2 + \cdots + (x_n-\bar{x})^2}{\cancel{n}\,(n-1)}}$

"SAMPLE SD"

1.5

$\Rightarrow$ CI

$\bar{x} \pm z\, s/\sqrt{n}$

$\bar{x} \pm z \sqrt{\dfrac{(x_1-\bar{x})^2 + \cdots + (x_n-\bar{x})^2}{n-1}} \Big/ \sqrt{n}$

EXAMPLE 1: $n = 3$ DATA: 2.1, 3.0, 1.87

s^2	x	$(x - \bar{x})^2$
	2.10	$\left(2.10 - \frac{6.97}{3}\right)^2$
	3.00	$(3.00 - \text{''})^2$
	1.87	$(1.87 - \text{''})^2$
TOTAL	6.97	TOTAL
AVG	6.97/3	TOTAL / (3 − 1)
$\bar{x}$		

$$s = \sqrt{\frac{\text{TOTAL}}{(3-1)}}.$$

EXAMPLE 2. DATA −1, 1

	x	$(x-0)^2$
	−1	$1 = (-1-0)^2$
	1	$1 = (1-0)^2$
TOT	0	2
AVG	0	$2/(2-1) = 2$

SO s (SAMPLE SD) IS $\sqrt{2}$

EXAMPLE 3. GIVEN SAMPLE OF $n = 50$

FINDING $\bar{x} = 37{,}492$, $s = 72{,}664$

$\Rightarrow$ 95% z-BASED CI for μ IS

$\bar{x} \pm 1.96\, s/\sqrt{n}$

$$\underbrace{37{,}492 \pm 1.96 \left(\frac{72{,}664}{\sqrt{50}}\right)}$$

$\pm\, 20{,}000^{+}$

WHAT IF WE SAMPLE WITHOUT REPLACEMENT?

$$\bar{x} \pm \overset{z}{1.96} \frac{\sigma}{\sqrt{n}} FPC$$

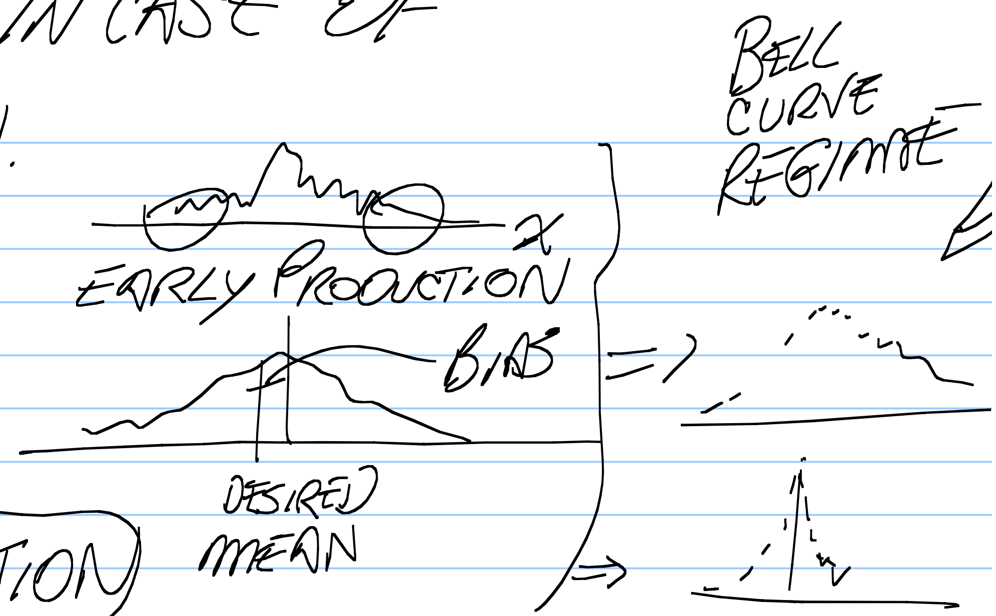
N = POPULATION SIZE

$$\bar{x} \pm z \, \sigma/\sqrt{n} \sqrt{\frac{N-n}{N-1}}$$

THE z-BASED CI REQUIRE LARGE SAMPLE SIZE (???)

HOW DO WE DEAL WITH SMALL n?

HEADWAY CAN BE MADE IN CASE OF A NORMAL POPULATION.



$\bar{x} \pm \cancel{z}\ \sigma/\sqrt{n}$ $\quad t$

$n > 1$

REQUIRE NORMAL POPULATION

EXAMPLE 4. APPENDIX D OF BOOK t-TABLE

DEGREES FREEDOM

THIS APP

$DF = n - 1$

∞	1.96
CONFIDENCE LEVELS	95%

SO GIVEN SAMPLE $n=5$ FROM A NORMAL POPULATION WITH $\bar{x} = 98.71$

SAY ALSO $s = .86$ $= \sqrt{\dfrac{(x_1 - 98.71)^2 + \cdots + (x_5 - 98.71)^2}{(5-1)}}$

$\Rightarrow$ YOUR 95% t-BASED CI FOR μ

IS $\bar{x} \pm t_4 \, s/\sqrt{n}$ $\quad 98.71 \pm \underbrace{2.776}_{t} \dfrac{.86}{\sqrt{5}}$

DF	
$5-1=4$	2.776
∞	1.96
CONF	95%

LARGER THAN WE'D LIKE BECAUSE s IS VOLATILE (2.776 GOES TO THIS) AND n = ONLY 5 SO $s/\sqrt{5}$ NOT SO SMALL.

CLAIM FOR t-CI

$$P\left(\mu \text{ IN } \boxed{\bar{x} \pm (2.776\ldots)\frac{s\ldots}{\sqrt{5}}}\right) = .95$$

FOR $n=5$ BUT LIKEWISE FOR EACH $n \geq 1$

eg. $n=30$ DF $= 30-1=29$

$$P\left(\mu \text{ IN } \boxed{\bar{x} \pm (2.045\ldots)\frac{s\ldots}{\sqrt{30}}}\right) = .95$$

AVG OF $n=30$

ASSIGNMENT #1

VERIFY μ, σ

(1a) z-BASED CI FOR μ:

1b. STANDARD ERROR FOR $\bar{x}$

(MEANS $\sigma_{\bar{x}}$)

WHICH IS $\sigma/\sqrt{n}$

$2.87/\sqrt{25}$ ANS.

μ = POPULATION MEAN OF x = LAST DIGIT OF STUDENT #

$$\mu = \frac{0+1+\cdots+9}{10} = 4.5$$

$$\sigma_{pop} = \sqrt{\frac{(0-4.5)^2+(1-4.5)^2+\cdots(9-4.5)^2}{10}}$$

$$= \sqrt{\frac{20.25+12.25+\cdots 20.25}{10}}$$

~ 2.87

1c. SAMPLE IS GIVEN $n = 25$ $\{3, 8, 9, 9, 1, \ldots\}$

CALC $\bar{x}$ = SAY $\bar{x} = 5.23$ (NOT)

$$s = \sqrt{\frac{(3-5.23)^2 + \cdots + (7-5.23)^2}{(25-1)}}$$ (SAY) $= 2.93$ (NOT)

z-BASED 95% CI FOR μ

I ASKED FOR THIS

$$x \pm z \, s/\sqrt{n}$$

$$5.23 \pm 1.96 \, \frac{2.93}{\sqrt{25}}$$ (NOT)