STT 200 §3.0 3-17-10a

Note Title 3/17/2010

Ch 24 TODAY + MONDAY.

PRIOR: 95% z-BASED CI for p: $\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}\hat{q}}}{\sqrt{n}}$

95% z-BASED CI for μ:

$\bar{x} \pm 1.96 \frac{s}{\sqrt{n}}$

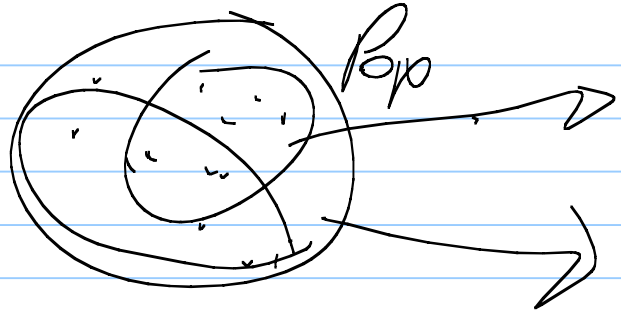
s = SAMPLE SD

$= \sqrt{\frac{(x_1 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n-1}}$

ACTUAL S.D. of $\hat{p}$: $\sigma_{\hat{p}} = \sqrt{pq}/\sqrt{n}$

ACTUAL S.D. of $\bar{x}$: $\sigma_{\bar{x}} = \frac{\sigma_{pop}}{\sqrt{n}}$

EXTEND ABOVE TO TWO POPULATIONS



SAMPLE 1 PLACEBO — p_1 = FRAC OF POP IMPROVING ON PLACEBO

SAMPLE 2 MED. — p_2 = " " " MED.

$\hat{p}_1$ $\hat{p}_2$ — $\hat{p}_1$ = FRAC SAMPLE 1 IMPROVING

$\hat{p}_2$ = FRAC " 2 "

USE $\hat{p}_1 - \hat{p}_2$ AS POINT EST OF $p_1 - p_2$

$\Rightarrow$ (GUESS) 95% z-BASED CI for $p_1 - p_2$

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96 \sqrt{p_1 q_1/n_1 \oplus p_2 q_2/n_2}$$

KNOW $\sigma_{\hat{p}_1} = \frac{\sqrt{p_1 q_1}}{\sqrt{n_1}}$

ALSO FOR 2

SO $\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1 q_1}{n_1} \oplus \frac{p_2 q_2}{n_2}}$

AROUND 95% OF ATTEMPTS THE ABOVE CI WILL COVER $p_1 - p_2$. (as $n_1 \to \infty, n_2 \to \infty$)

Eg. Suppose $n_1 = 100$ PLACEBO FINDING $\hat{p}_1 = .18$

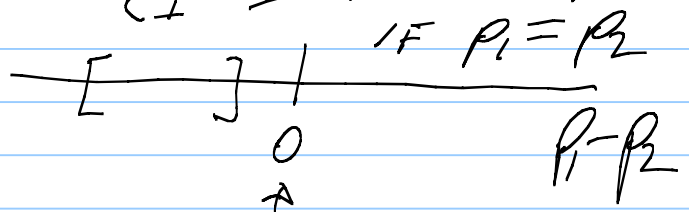
$n_2 = 50$ MED. " $\hat{p}_2 = .25$

95% CI for $p_1 - p_2$: $\hat{p}_1 \ominus \hat{p}_2 \pm 1.96 \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} \oplus \frac{\hat{p}_2\hat{q}_2}{n_2}}$

WORKS OUT TO $(.18 - .25) \pm 1.96 \sqrt{\frac{.18 \cdot .82}{100} \oplus \frac{.25 \cdot .75}{50}}$

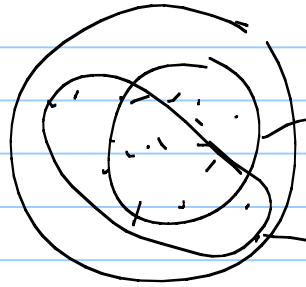
(WOULD WISH FOR 100 HERE)

CI — A .025 PR EVENT IF $p_1 = p_2$



$p_1 = p_2$ MED HAS NO EFFECT

SIMILAR EXTENSION FOR $\mu_1 - \mu_2$.



$n_1 = 100$ Simple n_1 scores $x_{11} \cdots x_{1n_1}$ PLACEBO $\quad \bar{x}_1 = 6.4 \quad s_1 = 8.3$

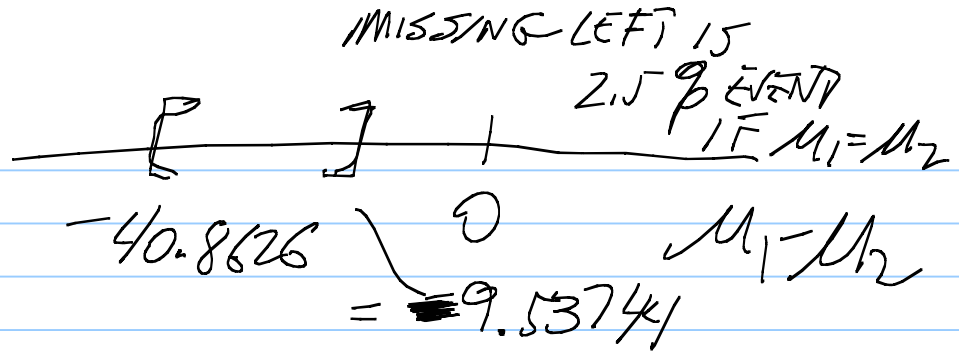
$n_2 = 50$ Simple n_2 scores MED $x_{21} \cdots x_{2n_2}$ $\quad \bar{x}_2 = 31.6 \quad s_2 = 56.2$

95% CI for $\mu_1 - \mu_2$:

$$(\bar{x}_1 - \bar{x}_2) \pm 1.96 \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(6.4 - 31.6) \pm 1.96 \sqrt{\frac{8.3^2}{100} + \frac{56.2^2}{50}}$$

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{\sigma_{pop1}^2}{n_1} + \frac{\sigma_{pop2}^2}{n_2}}$$



$\mu_1 - \mu_2 < 0$

PROTEIN HELPS