Answer Key

z-table use

1. Determine table entries for look up of $P(Z < 2.83)$.

a. row -2.83 column 0.00 b. row 2.83 column 0.00

c. row 2.8 column 0.03 d. row -2.8 column 0.03

2. Determine $P(Z < 2.83)$.

a. 0.9816 b. 0.9711 c. 0.9989 d. 0.9923 e. 0.9977

---

3. Closest table entry found when looking up of z satisfying $P(Z < z) = 0.846$.

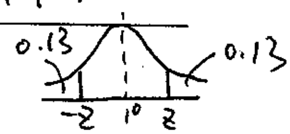
a. 0.8409 b. 0.8452 c. 0.8459 d. 0.8461 e. 0.8643

4. Value z satisfying $P(Z < z) = 0.86$ (use closest table entry). By reading from z-table, you'll get $z = 1.08$. So the closest answer is (b) 1.10.

a. ~~1.02~~ b. 1.10 c. 2.13 d. 0.94 e. 0.90

---

19 4. The z satisfying $P(Z > z) = 0.13$ also satisfies $P(Z < -z) = ?$



symmetric property

a. 0.18 b. 0.88 c. 0.09 d. 0.13 d. 0.22

5. Determine z satisfying $P(Z > z) = 0.13$ (use closest table entry). $P(Z < z) = 1 - 0.13 = 0.87$

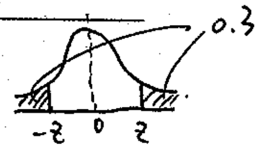
Use z-table to get z as 1.13

a. 0.77 b. 0.92 c. 0.84 d. 1.13 e. 1.01

---

6. The z satisfying $P(|Z| > z) = 0.30$ also satisfies $P(Z > z) = ?$

$= P(Z > z \text{ or } Z < -z)$



By symmetric property, $P(Z > z) = \frac{0.30}{2} = 0.15$

a. 0.30 b. 0.90 c. 0.15 d. 0.20 e. 0.10

CI for p in Bernoulli trials

7. What fraction of **97%** CI for p cover their intended target p?

a. around 0.85 b. around 0.97 c. exactly 0.97 d. exactly 0.985 e. ~1

$n=30$ $N=260$

8. A **with**-replacement sample of 30 hospital admittees from a population of 260 admittees finds 14 lack insurance for the recommended treatment. The CI for the corresponding population fraction p is:

$x=14$

a. $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}$ b. $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}\sqrt{\frac{N-n}{N-1}}$ c. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}\sqrt{\frac{N-n}{N-1}}$
d. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}$ e. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}$

Caution: d, e. are the same answers.

9. The **95%** CI for p (**not** 97% CI as in #7) is from #8 $\hat{p} = \frac{x}{n} = \frac{14}{30} \doteq 0.466667$

a. [0.498433, 0.534900] b. [0.102523, 0.83081] c. [0.288142, 0.645191]
d. [0.357366, 0.575967] e. [0.202523, 0.73081]

$0.466667 \pm 1.96 \times \sqrt{\frac{0.466667 \times 0.53333}{30}}$
$= 0.466667 \pm 0.178525$
$\Rightarrow [0.288142, 0.645192]$

10. The Agresti-Coull estimate of p (different from $\hat{p}$) is $\tilde{p}$ equal to $\tilde{p} = \frac{x+2}{n+4} = \frac{16}{34} \doteq 0.470588$

a. 0.340909 b. 0.220395 c. 0.199699 d. 0.470588 e. 0.254910

11. The Agresti-Coull 95% CI for p is $\tilde{p} \pm z \cdot \sqrt{\frac{\tilde{p}\tilde{q}}{n+4}} = 0.470588 \pm 1.96 \times \sqrt{\frac{0.470588 \times 0.529412}{34}}$

a. [0.200847, 0.480972] b. [0.302811, 0.638366] c. [0.288142, 0.645191]
d. [0.357366, 0.575967] e. [0.202523, 0.73081]

$= 0.470588 \pm 0.167778$
$\Rightarrow [0.302810, 0.638366]$

$n=30$ $N=260$

12. A **without**-replacement sample of 30 hospital admittees from a population of 260 admittees finds 14 lack insurance for the recommended treatment. The form of a **95%** CI for the corresponding population fraction p is:

$x=14$

a. $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}$ b. $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}\sqrt{\frac{N-n}{N-1}}$ c. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}$

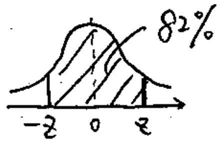
d. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}$ e. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}\sqrt{\frac{N-n}{N-1}}$

Caution: c, d are the same.

13. The above **95%** CI for p is

a. [0.298433, 0.634900] b. [0.302811, 0.638366] c. [0.29702, 0.636313]
d. [0.357366, 0.575967] e. [0.202523, 0.73081]

14. For a 82% CI what z would be used?



$P(Z<z) = 0.82 + \frac{1-0.82}{2}$
$= 0.91$
Use z-table to get z as 1.34

a. 1.45 b. 1.65 c. 1.34 d. 1.25 e. 1.75

tests of hypotheses

15. It is desired to test the null hypothesis p = 0.2 versus the alternative p > 0.2. where p is the fraction of snow blower sales having a service contract in the deal. The rate 0.2 applied just prior to a new advertising "rollout" promoting the service contract. Give the form of the z-test statistic. Note the following:

This is not a CI.
The test is not being set up in terms of X.

a. $\dfrac{\hat{p}-p_0}{\sqrt{n\,p_0\,q_0}}$ b. $\dfrac{\hat{p}-p_0}{\sqrt{p_0\,q_0/n}}$ c. $\dfrac{\hat{p}-p_0}{\sqrt{\hat{p}\,\hat{q}/n}}$ d. $\dfrac{\hat{p}-p_0}{\sqrt{n\,\hat{p}\,\hat{q}}}$ e. $\hat{p} - p_0$

16. A sample of 100 ($n=100$) snow blower sales finds 25 ($X=25$) had the contract as part of the deal. Determine the P-value for this data employing your choice in #15.

a. 0.04010 b. 0.02213 c. 0.03181 d. 0.1056 e. 0.09327

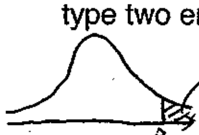
$p\text{-value} = P\left(z > \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}\right) = P\left(z > \frac{25/100 - 0.2}{\sqrt{\frac{0.2\times0.8}{100}}}\right)$
$= P(z > 1.25) \doteq 0.1056$

tests with pre-assigned probabilities of type 1 and type 2 error

17. It is desired to test

H_0: p = 0.2 type one error probability 0.04
H_A: p = 0.4 type two error probability 0.01

Determine Z_0.



type 1 error probability 0.04

$P(Z > z_0) = 0.04 \Rightarrow P(Z < z_0) = 0.96$
Use z-table to get z_0 as 1.75

a. 2.59 b. 1.75 c. 2.33 d. 2.67 e. 2.81

18. You are given that Z_1= -2.33. Using the information in #17 determine the sample size n required to achieve the goals set out above.

a. 231 b. 81 c. 135 d. 85 e. 78

c

$n = \left(\frac{\sqrt{0.2\times0.8}\times|1.75| + \sqrt{0.4\times0.6}\times|-2.33|}{0.2-0.4}\right)^2$

$= 84.7746$ ↑ round it up to 85

$\sqrt{(N-n)/(N-1)}$

$\sqrt{npq}$

$\sqrt{pq/n}$

$\sqrt{\hat{p}\hat{q}/n}$

$$\left(\frac{\sqrt{p_0\,q_0}\,\left|z_0\right| + \sqrt{p_1\,q_1}\,\left|z_1\right|}{p_0 - p_1}\right)^2$$

$$z_0\sqrt{n\,p_0\,q_0} + 0.5 + n\,p_0$$