Recitation assignment due in recitation 2 - 2 - 10. See chapters 16 and 17.

Reminder: Exam 1 is 2-3-10. Lecture 1-27-10 will be very important in getting you started with the material below.

Errata:

On page 426 it is said that "410, 420. A random variable that can take any numeric value within a range of values is called a continuous random variable." This is incorrect. To see the difficulty imagine that I flip a coin and if heads occurs I say \( X = 1 \), otherwise I spin a pointer and announce \( X \) as the angle, in infinite precision degrees 0 to 360, at which the pointer comes to rest. \( X \) may take any value in the interval \([0, 360]\) but is not continuous since there is discrete probability 0.5 that \( X \) takes the value 1. See lecture notes 1-27-10.

On page 426 it is said that "410. The probability model is a function that associates a probability \( P \) with each value of a discrete random variable \( X \), denoted \( P(X = x) \), or with any interval of values of a continuous random variable." They should have said "The probability model for a random variable \( X \) . . " To see the difficulty consider the probability model usually specified for the toss of a coin which consists of the set \( \{ H, T \} \) of possible outcomes having respective probabilities \( (0.5, 0.5) \). If you code the model numerically such as outcomes \( \{ 0 \) for \( T \), 1 for \( H \} \) with respective probabilities \( (0.5, 0.5) \) you have another probability model whose possible outcomes \( \{ 0, 1 \} \) are numeric. This model is what the authors are talking about in 410. This model is also called the probability distribution of random variable \( X \). We must not make the oversight of ruling out \( \{ H, T \}, (0.5, 0.5) \) as a probability model and we must recognize that we are talking about a probability distribution on the real line commonly referred to as the probability distribution of a random variable \( X \).

1-12. Random variable \( X \) has the following distribution (some useful calculations are shown):

\[
\begin{array}{cccccc}
 x & p(x) & x^2 & (x - (-0.7))^2 & p(x) & x^2 p(x) \\
 0 & 0.2 & 0 & 0.49 & 0.2 & 0 & 0.2 \\
 1 & 0.3 & 0.3 & 2.89 & 0.3 & 1 & 0.3 \\
 -2 & 0.5 & -1.0 & 1.69 & 0.5 & 4 & 0.5 \\
 \text{totals} & 1.0 & -0.7 & 1.81 & 2.3 & \\
\end{array}
\]

Confirm all calculations of the table before proceeding.
1. Determine $E(X) =$
   (see pg. 411)

2. Directly read Variance $X$ from the table =
   (see pg. 413)

3. Calculate $V(X) = E(X^2) - (E(X))^2 =$
   (see that your answer agrees with #2)

4. Determine $E(3X + 7) =$
   (see pp. 414-415)

5. Determine $E(3X - X - 6) =$

6. Determine Variance of $(3X + 7) =$
   (use rules, see pg. 415)

7. Determine Variance of $(3X - X + 7) =$
   (merge $3X$ with $-X$ first)

8. If $Y$ is a random variable with $E(Y) = 6$ determine $E(X - 2Y + 4) =$
   (use rules, see pg. 415)

9. If $Y$ is a random variable independent of $X$ with Variance $Y = 2$ determine Variance of $(5X - Y + 11) =$
   (see pg. 416)

10. Determine the standard deviation of random variable $X$ (written SD or "sigma" or $\sigma$, or $\sigma_X$ when we wish to specify which random variable it is the standard deviation of.
    (see pg. 426)

11. Determine the SD of random variable $(3X - 2X + 4) =$
    (use the rules, see pg. 426, be careful since $3X$ and $X$ are not independent!)

12. Refer to #9. Determine the standard deviation of $(5X - Y + 11)$. 
    (use rules, see pg. 426)
13-17. Lottery 1 returns random variable $X$ having expectation 17 and variance 4. Lottery 2 returns random variable $Y$ having expectation 30 and variance 9. We are invited to play each of these but it will cost us 2 to play for $X$ and 3 to play for $Y$. As a bonus, we will earn 1.4 $X$ instead of $X$.

13. In terms of $X$, $Y$, 2, 3, 1.4 express a random variable describing our actual NET return $R$ if we accept the offer.

14. Using the rules determine $E(R) = $ 

15. Using the rules determine Variance of $R$ if $X$, $Y$ are independent.

16. From #15 give the SD of $R = $ 

17. If each of $X$, $Y$ follows a normal distribution the so will $R$ (provided $X$, $Y$ are independent, see pg. 422). Assuming that $R$ follows a normal distribution sketch the distribution with the mean and SD in place and determine a 68\% Interval around the mean.

Refer to chapter 17. Before exam 1 we will cover only the definition of Bernoulli Trials, the binomial distribution and its normal approximation, and the Poisson distribution and its normal approximation.

18-22. A fair coin will be tossed 100 times (any H counts as a "success").

18. What are the number $n$ and probability $p$ of Bernoulli trials? (see pg. 433)

19. Define random variable $X$ as the number of heads seen in executing #18. Determine $E(X) = $, $Variance\ X = $, $SD\ X = $ 

20. Verify the condition (pg. 439) whereby we may approximate the distribution of $X$ by a normal and sketch the normal approximation of the distribution of $X$ labels included.
21. Use #20 to fill out the following:
   \[ P( X \text{ falls in the range } [ , ] ) \sim 0.68 \]
   \[ P( X \text{ falls in the range } [ , ] ) \sim 0.95 \]
   \[ P( \text{ we get between 40 and 55 heads in 100 tosses } ) \sim \]

22. Determine the approximate 68% interval for the number of heads in 10,000 tosses of a fair coin.

23-27. A fair six-sided die will be tossed 100 times (any "ace", i.e. face turning up, will be counted as a "success").

23. What are the number n and probability p of Bernoulli trials?
   (see pg. 433)

24. Define random variable X as the number of aces seen in executing #23. Determine \( E \ X = \), \( \text{Variance } X = \), \( \text{SD } X = \)

25. Verify the condition (pg. 439) whereby we may approximate the distribution of X by a normal and sketch the normal approximation of the distribution of X labels included.

26. Use #25 to fill out the following:
   \[ P( X \text{ falls in the range } [ , ] ) \sim 0.68 \]
   \[ P( X \text{ falls in the range } [ , ] ) \sim 0.95 \]

27. Determine the approximate 68% interval for the number of aces in 10,000 tosses of a fair die.
28-31. A hospital averages around 4.7 emergency admissions for eye injury per night. Past experience indicates that these counts $X$ of rare events are acceptably modelled by the Poisson distribution.

28. Determine $E X =$

29. Determine $SD X =$

30. Since $E X \geq 3$, sketch the normal distribution approximating the distribution of $X = \#$ admitted with eye injury in a given night. Label mean and SD.

31. Determine a 95% interval for $X$. Would you be surprised to see so many as 9 admissions for eye injury in a given night? Why?