NORMAL DISTRIBUTIONS, Z-TABLE p.210,
ALL NORMALS ARE ALIKE IN 6 UNITS FROM THE MEAN.
STANDARD SCORES.

IQ NORMAL MEAN 100 SD 6 = 15.

50  P(IQ IN 85 TO 115) ~ .68

P(IQ IN 100 TO 115) ~ .68/2 = .34

.68  P(IQ IN 70 TO 130) ~ .95 (Rule of Thumb)

P(IQ > 130) ~ .025
TABLE: \( \mu = 0 \quad \sigma = 1 \) STANDARD NORMAL

\[
\begin{array}{ccc}
0 & 1.00 & \frac{3}{3} \\
& & 1.0 \boxed{0.3413} \\
& & \frac{3}{3} \\
& & \frac{689}{5} \text{ SHOULD REALLY BE 68.26%}
\end{array}
\]

Closer to 0.6826
9. Find $P(Z < 1.83)$ (Use Table Z)

\[ = 0.5 + 0.4664 = 0.9664 \]

9. $P(Z > -1.83)$

\[ = .9 \]

\[ -1.83 \]

3. Scores: IQ $M = 100$, $\sigma = 15$

$P$(IQ in range 100 to 123) = $P$(Z in $\frac{100-100}{15}$ to $\frac{123-100}{15}$)

IQ 100 → $Z = \frac{100-100}{15} = 0$

Raw 123 → $Z = \frac{123-100}{15} = 1.53$
\[ P(\text{IQ} \in 100 \pm 1.53) = P(Z \in 0 \text{ to } 1.53) \]

\[ 1.5 \approx 1.5 \]

\[ 0.4370 \]

So, \( P(\text{IQ} < 123) = 0.5 + 0.4370 = 0.937 \)

Someone with IQ = 123 is almost at 94th percentile of IQ.

**Normal Approx of Binomial**

Eqn: \( X = \# H \text{ in } 100 \text{ tosses of a coin, } 100 \text{ times} \)

\[ E(X) = np = 100 \left( \frac{1}{2} \right) = 50 \]

\[ \sigma_X = \sqrt{np(1-p)} = \sqrt{100 \times \frac{1}{2} \times \frac{1}{2}} = 5 \]
If $x = 5.5$, then $z = \frac{5.5 - 5.0}{0.5} = 1$

$P(\#H \text{ in } 50 \pm 5) \approx P(\frac{49.5 - 50}{5} \text{ to } \frac{55.5 - 50}{5})$

$z = P(\frac{49.5}{5} \text{ to } \frac{55.5}{5})$

$z = P(2 \text{ in } 0.10 \text{ to } 1.10)$

$P(\#H \text{ in } 50 \pm 5) = 0.039 + 0.3643 = 0.4033$

**BATTING**

$p = 0.3 \quad n = 100 \quad \mu = np = 30 \quad \sigma = \sqrt{100 \cdot 0.3 \cdot 0.7}$
$\sigma = \sqrt{np(1-p)} = 4.58$

$\sigma = 4.58 \text{ NOT SO FAR FROM } \sqrt{5}$

$\mu = mp = 30$

$30 \pm 4.58 \approx 68\% \text{ INTERVAL}$

$\text{Actual } P( \text{#HITS IN 27 to 34})$

$\approx P(Z \text{ in } \frac{26.5-30}{4.58} \text{ to } \frac{34.5-30}{4.58}) \approx 0.61$

Why did I not go to 26 to 34 - BLIPPED!

Binomial $n$ LARGE $p$ SMALL
For $n = 500$, $p = \frac{1}{100}$, $M = np = 500 \cdot \frac{1}{100} = 5$

$$p(3) = \frac{500!}{3! \cdot 497!} \left( \frac{1}{100} \right)^3 \left( \frac{99}{100} \right)^{497}$$

For $n = 5000$, $p = \frac{1}{1000}$, $M = np = 5000 \cdot \frac{1}{1000} = 5$

$$p(3) = \frac{5000!}{3! \cdot 497!} \left( \frac{1}{1000} \right)^3 \left( \frac{999}{1000} \right)^{497}$$

$\sqrt{np(1-p)} \approx \sqrt{np}$

Very close to one another

And $\sim e^{-5} \frac{5^3}{3!}$

$e \sim 2.718281828...$

Form $e^{-\mu} \frac{\mu^x}{x!}$ for $x = 0, 1, 2, ...$ Poisson
Poisson Raisin Cookies.

Make 144 Cookies

Your Cookie

\[ 5 \times 6 = 4 \times (144) \]

\[ \rho = \frac{1}{144} \quad n_1 = 5 > 6 \]

\[ np = 5 \times 6 \left( \frac{1}{144} \right) = 4 \]

\[ \rho(3) = e^{-\mu} \frac{\mu^3}{3!} = e^{-4} \frac{4^3}{6} \]

\[ 3! = 3 \times 2 \times 1 = 6 \]

\[ \rho(0) = e^{-4} \frac{4^0}{0!} = e^{-4} = 0.018 \]

Comment: You may justify this application of Poisson by baking a lot of cookies, finding

~ 29% have no raisins, ~ 20% have 3 raisins, etc.
NORMAL APPROX OF POISSON:

\[ p(x) = e^{-\mu} \frac{\mu^x}{x!} \]

\( \mu \) - MEAN

\( \sigma = \sqrt{\mu} \)

APPLICABLE (THIS COURSE) \( \mu \geq 10 \)

Say million oatmeal cookies (9" diameter) say avg # m&m per cookie is 16

\[ x = \# \text{ of m&m in a given cookie} \]

\[ \sqrt{16} = \sqrt{\mu} \text{ for Poisson} \]

\[ x = \# \text{ m&m} \]
BEAUTIFUL! WE ONLY NEED M!!

If we AVG 25 CLAIMS/DAY ~ Dist 25

x = # CLAIMS