Today, use normal distributions as they are - e.g., in natural world measurements, business activities, science (forced by quality assurance methodology), approx of other distributions.

First normal distributions, Z-scores, Table pg. 210.

Look at Table.

\[ Z = \frac{X - \mu}{\sigma} \]

68% 100 ± 1

1.0 34.13

3.00 0.02

0 ± 1

\[ Z = 1.00 \]

\[ Z = 3.4 \]

Note: e.g., IQ \( \mu = 100 \)

\( \sigma = 1 \)
So \( P(\text{IQ in range 100 to 115}) \approx \frac{.3413}{3} \)

Likewise

\[
\begin{array}{c}
\text{z} & 0.0625 \\
1.9 & 0.4750
\end{array}
\]

Another use of Table:

Find \( P(\text{IQ} < 123) = P\left(z < \frac{123 - 100}{15}\right) = P\left(z < \frac{23}{15}\right) = P\left(z < 1.53\right) \)

\(0.5\) Table gives \( \frac{3}{15} = 0.03 \implies 1.5 [0.437] \implies 0.5 + 0.437 = 0.937 \)
Yet another example of table use.

\[ P(Z > 2.04) = .5 - .4793 = .0207 \]

Binomial is approximated by normal \( n \approx mp \)

So \( m = 100, p = \frac{1}{2} \) (100 coin tosses) \( \sigma = \sqrt{np(1-p)} \)

\[
\mu = 100 \left( \frac{1}{2} \right) = 50 \quad \sigma = \sqrt{100 \left( \frac{1}{2} \right) \left( \frac{1}{2} \right)} = 5
\]

True \( \left( \frac{1}{2} \right)^{100} \) \( \frac{100!}{50!50!} \left( \frac{1}{2} \right)^{100} = \frac{1}{100} \)

Discrete sum of probabilities = 1

\[ 0.45 \to 0.5 \to 100 \quad X = \#4 \]

Continuous area = 1

~.68 later do more accurately
Another approach:

\[ P(\text{ge exactly 10 H}) = P(\text{continuous between 9.5 and 10.5}) \]

\[ P(50) = \frac{100!}{50!50!} \left( \frac{1}{2} \right)^{10} \]

\[ = \frac{100 \cdot 99 \cdot \ldots \cdot 52}{50 \cdot 49 \cdot \ldots \cdot 50} \frac{1}{1024} \approx 0.0398 \]

\[ = 2 \times 0.0398 = 0.0796 \]

Cool! Top level thinking!
Pause to look at #1 from HW 7-19-10

\text{GIVE NORMAL } \mu = 1.77 \quad \sigma = 0.9

(c) \text{ STDEV SCORE OF 1.8 is } z = \frac{1.8 - 1.77}{0.9} = 0.33

\text{Poisson Distribution for counts of rare events.}

\text{And its approximation by Normal.}

\text{Think first of Binomial} \quad p(x) = \frac{\mu^x}{x!} \cdot p^x (1-p)^{\mu-x}

\begin{align*}
\mu \text{ large} & \Rightarrow p(x) \approx \text{ depends only on } np \\
\mu \text{ small} & \Rightarrow p(x) \approx \text{ depends only on } np
\end{align*}
TRY 500 TIMES (n = 500) WITH p = \frac{1}{100} \text{ EXP: np = 5, on AVE.}

\[ p(3) = \frac{500!}{3!} \frac{1^3 (99)^4}{100^3} \frac{99}{100} \]

\[ \alpha = 3 \]

INSTEAD TRY n = 5000, p = \frac{1}{1000} \text{ EXP: np = 5000 \left( \frac{1}{1000} \right) = 5}

\[ p(3) = \frac{5000!}{3!} \frac{1^3 (999)^4}{1000^3} \frac{999}{1000} \]

\[ \text{Almost the same!} \]

\[ \approx e^{-mp} \frac{(mp)^x}{x!} \]

\[ e = 2.718281828 \]

\[ = e^{-5} \frac{5^3}{3!} = e^{-5} \frac{125}{6} \]
Example. We are 2.3 empty seats per flight (experience)

If Poisson is applicable

\[ P(\text{given flight has 3 empty seats}) \]

\[ \approx e^{-2.3} \frac{(2.3)^3}{3!} \]

Example. Cookies.

144 Baker's dozen

6 \times 144 Raisins. = 864

Dough

\[ p = \frac{1}{144} \]

\[ n = 864 \]

\[ mp = (\text{wanted}) \]

3! = 3 \times 2 \times 1 = 6
\[ P(\text{given cookie has 3 raisins}) \approx e^{-\frac{6}{3}} = e^{-2} = 0.135 \]

\[ P(\text{given cookie has 0 raisins}) = e^{-6} \frac{6^0}{0!} = e^{-6}/0! = 0.002\]

\[ \text{Example: we ave 4.8 lightning per season} \]

\[ P(\text{get none this season}) \approx e^{-4.8} \frac{4.8^0}{0!} = e^{-4.8} = 0.01\]

\[ \text{Normal approx of Poisson} \]
IF MEAN OF POISSON IS $\geq 10$ (say) CALL IT M

POISSON $\sim$ NORMAL

WHY $\sqrt{M}$?

THINK $\sqrt{M} \cdot p(1-p)$ RARE EVENTS

$\sim \sqrt{M} \cdot p \approx 0$

EXAMPLE: NORMAL APPROX. OF POISSON $\mu \geq 10$ RULE WE INVoke

Suppose we ave 16 oil spill incidents per year.

Think Poisson because it deals with counts of rare events may be applicable. - Supposition! Check later
$\sqrt{m} = 4$

$\sim 68\% \text{ CHANCE}$

CRUDE GET APPROX 12 TO 20.

$m = 16 \sim 20.5$ MORE ACCURATE

NORMAL APPROX

0, 2, 4, 6, 8, 10, 12, 14, 16, 20, 25, 30, 35, 40