INSTRUCTION TO SUM

\[
\bar{X} = \frac{\sum X_n}{n} = \frac{X_1 + \cdots + X_n}{n}
\]

FOR (IS) ABOVE \( M = \frac{1 + (-1)}{2} = 0 \)

THINK \( H = 1 \) \( T = -1 \) POSSESSES OF FAIR COIN

\( \begin{array}{ccccccc}
& H & H & H & T & H & H \\
\end{array} \)

LIKE SELECTING \( 1\ 1\ 1\ 1\ -1\ 1\ 1\ )\) IN WITH-REPLACEMENT SELECTIONS FROM (IS) \( \{1, -1\} \).
**STANDARD DEVIATION \( \sigma \) OF A LIST.**

**FIRST NOTION OF VARIANCE \( \sigma^2 \)**

**DEFINITION:**

\[
\text{VARIANCE} = \sigma^2 = \frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \ldots + (x_m - \mu)^2}{m}
\]

\( x_i - \mu = \text{DEVIATION of } i^{\text{TH}} \text{ item in the list from } \mu \)

For little list \( \{1, -1\} \):

\[
\mu = \frac{1 + (-1)}{2} = 0
\]

\[
\sigma^2 = \frac{(1-0)^2 + (-1-0)^2}{2} = \frac{2}{2} = 1
\]

\[\Rightarrow \sigma = \sqrt{\sigma^2} = \sqrt{1} = 1 \text{ as root.} \]
Example: Die \[ \{1, 2, 3, 4, 5, 6\} \] \[ n = 6 \]

\[ \mu = \frac{1+2+3+4+5+6}{6} = 21 = 3.5 \]

\[ \sigma^2 = \frac{(1-3.5)^2 + (2-3.5)^2 + \ldots + (6-3.5)^2}{6} \]

\[ = \frac{6.25 + 2.25 + 1.25 + 0.25 + 0.25 + 6.25}{6} \]

\[ (-2.5)^2 = 6.25 \quad (-.5)^2 = .25 \quad (-1.5)^2 = 2.25 \quad (.5)^2 = .25 \]

\[ \Rightarrow \sigma = \sqrt{\frac{17.5}{6}} = 1.707 \]

What is \( \sigma \) good for?

Repeatingly toss fair die \[ \] ? (Correct?)

Examine the growth of sum of tosses.
Smoke Plume

Slight wind.

\[ \mu = 0 \text{ trend line is horizontal} \]

Conclusion: \( \sigma \) has its hand on the behavior of randomness.
At 10,000 plays

\[ \sim 1.707(140) \sim 300 \]

\[ \frac{(1.707)}{\text{Die 6}} \sqrt{10000} \]  

(Casino's Avg Take)

3.5(10000) = 35,000

Chance is highly regular in the aggregate.

VARIABILITIES IN CHANCE OUTCOMES ARE RELATED TO 0. WITH AGGREGATE BEHAVIOR THE VARIABILITY FROM AVG MATTERS RELATIVELY LESS.

PROPERTIES OF VARIANCE, STD DEV, AND M.
TWO LOTTERIES (GAMES)

DIE \(1, 2, 3, 4, 5, 6\) \(M = 3.5\)

\(\sigma = 1.707\)

OR "OTHER DIE" \(1, 1, 1, 6, 6, 6\) \(M = 3.5\)

\(\sigma = 2.5\)

\[
\sigma = \sqrt{\frac{(1-3.5)^2 + (1-3.5)^2 + (1-3.5)^2 + (6-3.5)^2 + (6-3.5)^2 + (6-3.5)^2}{6}}
\]

\[
\sigma = \sqrt{\frac{6.25 + 6.25 + 6.25}{6}} = 6.25
\]

\[
\sigma = \sqrt{6.25} = 2.5
\]

\[
(2.5)^2 = 6.25
\]
TOTAL RETURN

OTHER DIE 2.5 V2 # LOG #

3.5 (#PLAYS)

1.707 V2 # LOG #

LOG_e = ln Key

LINEARITY OF M.

\[ M_{x+b} = \frac{x_1 + b + x_2 + b + \cdots + x_n + b}{n} = M_x + b \]

so \[ M_{x+b} = M_x + b \]
Also \[ M_{\text{max}} = \frac{\Theta x_1 + \Theta x_2 + \cdots + \Theta x_n}{n} = a M_x \]

so \[ M_{\text{max}} = a M_x \]

**Couple THESE** \[ M_{ax+b} = M_{\text{max}} + b = a M_x + b \]

so \[ M_{ax+b} = a M_x + b \]

**Scale** \[ \text{Change} \quad \text{Location Change} \]

**Example** \[ x = 1157 \text{ OF } \] °F temps \[ \text{K} \text{now} C^\circ = \frac{5}{9} (F - 32) \]

so if \[ M_x = 59.3^\circ F \] \[ \Rightarrow \begin{bmatrix} M_x = \frac{5}{9} (59.3 - 32) \\
\frac{5}{9}(x-32) \end{bmatrix} \]
\[ \text{Also} \quad \sigma^2_{ax} = a^2 \sigma^2_x \quad \Rightarrow \quad \sigma_{ax} = |a| \sigma_x \]

**Couple THESE:**

\[ \sigma^2_{ax+b} = a^2 \sigma^2_x \quad \Rightarrow \quad \sigma_{ax+b} = |a| \sigma_x \]