Note Title 7/6/2010

STT 200 SECTION 202 JULY 7

LIST OF NUMBERS $\{-1, 1\}$.

INSTRUCTION TO SUM

MEAN (AVG) LIST $x_1, x_2, \ldots, x_n$

CAP SIGMA

$$\mu = \frac{x_1 + \cdots + x_n}{n} = \sum_{i=1}^{n} x_i / n$$

"mu"

FOR LIST ABOVE $\mu = \frac{1+(-1)}{2} = 0$

THINK $H \equiv 1$ $T \equiv -1$ TOSSES OF FAIR COIN

eg H H H H T H H

LIKE SELECTING 1 1 1 1 -1 1 1

IN WITH-REPLACEMENT SELECTIONS

FROM LIST $\{1, -1\}$.

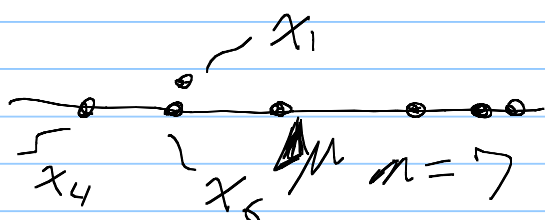
STANDARD DEVIATION σ OF A LIST.

σ LOWER CASE SIGMA

FIRST NOTION OF VARIANCE σ^2

DEFINITION:

$$\boxed{\text{VARIANCE} = \sigma^2 = \frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}}$$



$x_i - \mu$ = DEVIATION OF i^{TH} ON THE LIST FROM μ

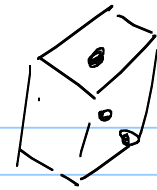
FOR LITTLE LIST $\{1, -1\}$.

$$\mu = \frac{1+(-1)}{2} = 0$$

$$\sigma^2 = \frac{(1-0)^2 + (-1-0)^2}{2} = \frac{2}{2} = 1$$

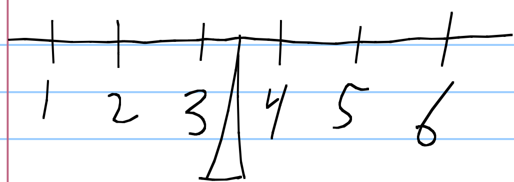
$$\Rightarrow \sigma = \sqrt{\sigma^2} = \sqrt{1} = 1 \text{ POS ROOT.}$$

EXAMPLE: DIE



LIST $\{1, 2, 3, 4, 5, 6\}$. $n = 6$

$$\mu = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2} = 3.5$$



$$\sigma^2 = \left((1-3.5)^2 + (2-3.5)^2 + \cdots + (6-3.5)^2\right)/6$$

$$= (6.25 + 2.25 + .25 + .25 + 2.25 + 6.25)/6$$

$(-2.5)^2 = 6.25$ $\quad (-.5)^2 = .25$

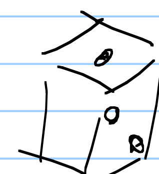
$(-1.5)^2 = 2.25$ $\quad (.5)^2 = .25$

$$= 17.5/6 = 2.9166$$

$$\Rightarrow \sigma = \sqrt{17.5/6} = 1.707$$

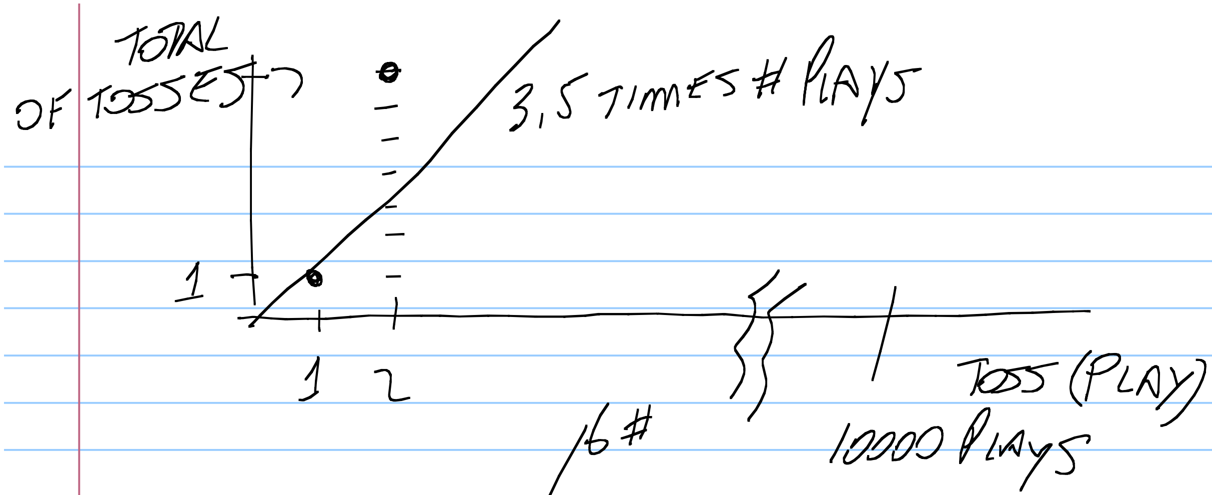
WHAT IS σ GOOD FOR?

REPEATEDLY TOSS FAIR DIE



? CORRECT?

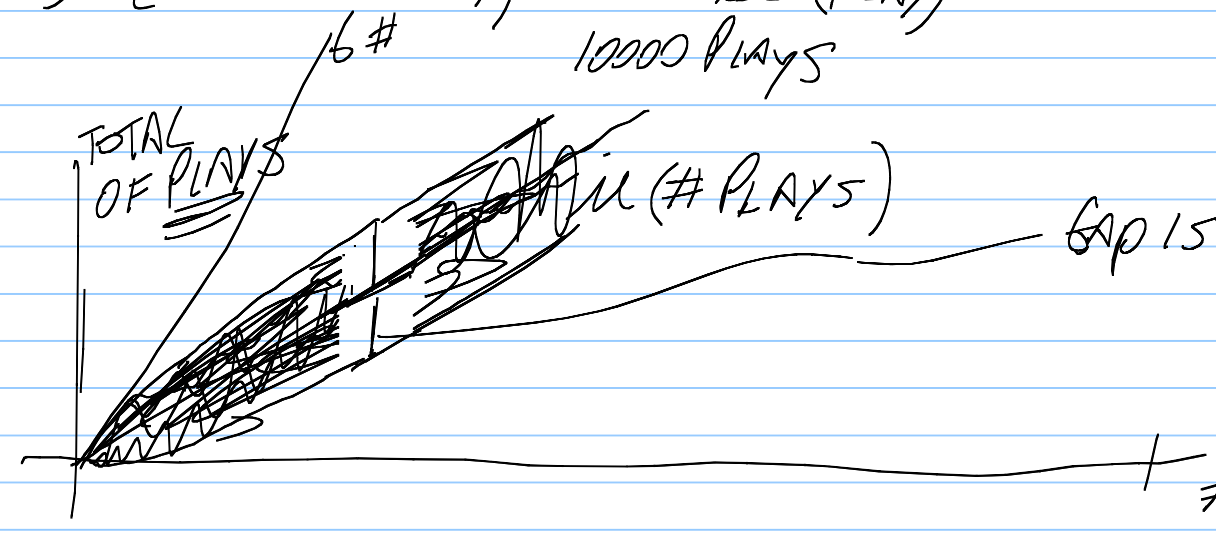
EXAMINE THE GROWTH OF SUM OF TOSSES.



GAME:

μ (EACH PLAY) = 3.5

σ (") = 1.707



GAP IS $\sigma_{DIE} \sqrt{2 \# \log_e \log_e \#}$

$\# = \#$ PLAYS,

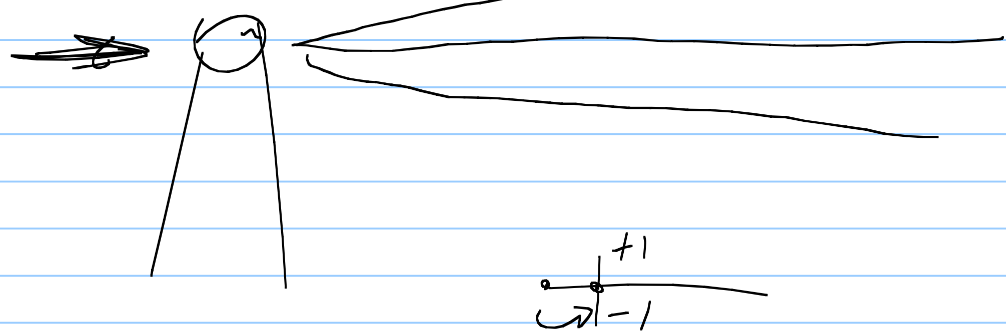
LOTS OF PLAYERS!!

— TRAJECTORIES.

SMOKE PLUME

SAME PHENOMENON.

SLIGHT WIND.

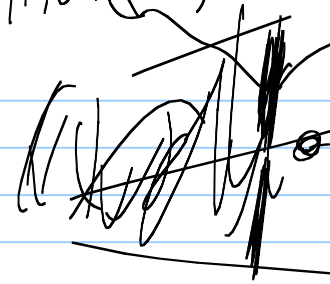


$\mu = 0$ TREND LINE IS HORIZONTAL

CONCLUSION: σ HAS ITS HAND ON THE BEHAVIOR OF RANDOMNESS.

AT 10,000 PLAYS

$\approx 1.707(140) \sim 300$



ALL PLAYERS IN RANGE

$\left(\frac{1.707}{\text{DIE } \sigma}\right)\sqrt{2 \cdot 10000 \log_e \log_e 10000}$

CASINO'S AVG TAKE

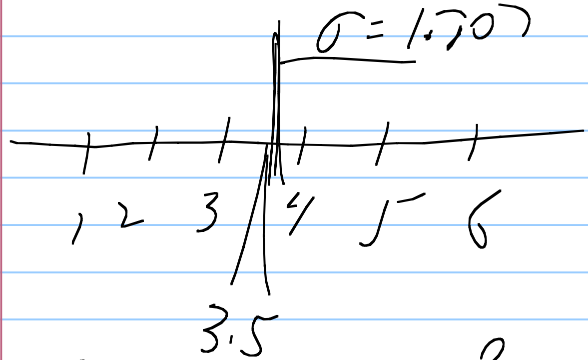
$3.5(10000) = 35{,}000$

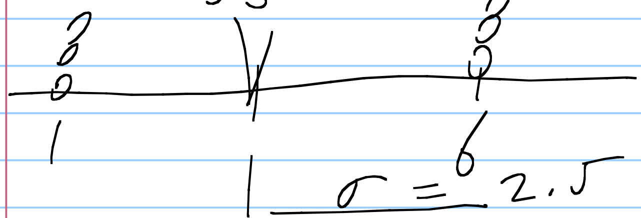
CHANCE IS HIGHLY REGULAR IN THE AGGREGATE.

VARIABILITIES IN CHANCE OUTCOMES ARE RELATED TO σ. WITH AGGREGATE BEHAVIOR THE VARIABILITY FROM AVG MATTERS RELATIVELY LESS.

PROPERTIES OF VARIANCE, STD DEV, AND μ.

TWO LOTTERIES (GAMES)





DIE 1, 2, 3, 4, 5, 6 $\mu = 3.5$

$\sigma = 1.707$

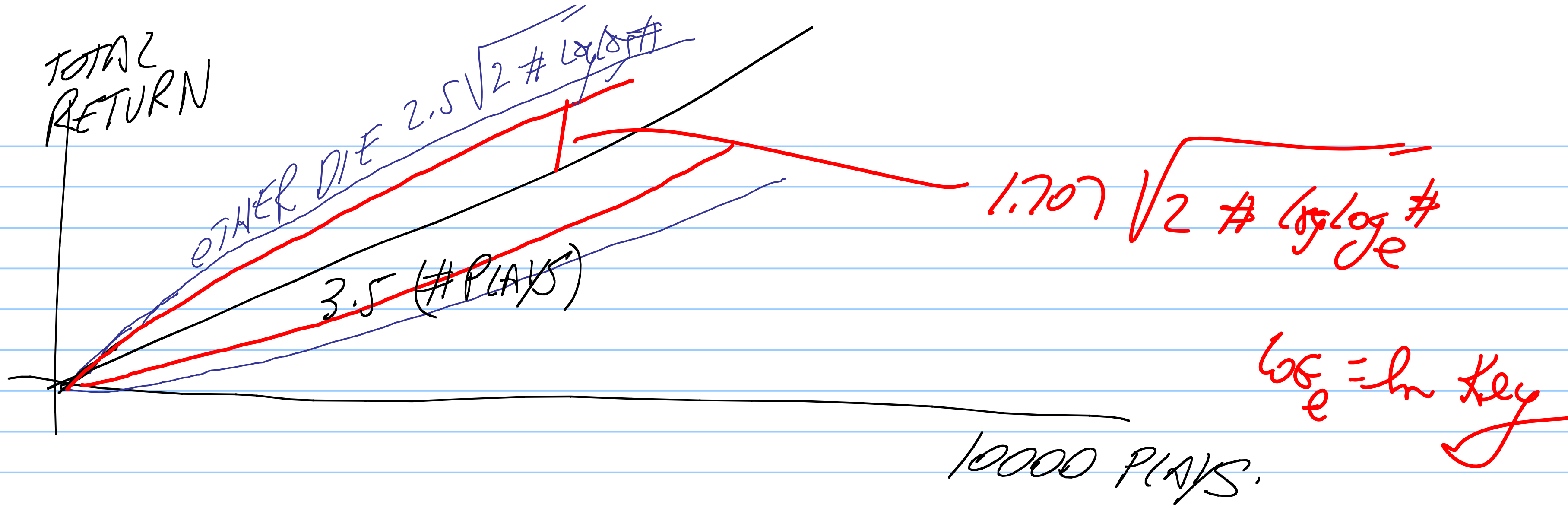
OR "OTHER DIE" 1, 1, 1, 6, 6, 6 $\mu = 3.5$

$\sigma = 2.5$

$$\sigma^2 = \left[(1-3.5)^2 + (1-3.5)^2 + (1-3.5)^2 + (6-3.5)^2 + (6-3.5)^2 + (6-3.5)^2\right] / 6$$

$$= (6.25 + 6.25 + \cdot + 6.25)/6 = 6.25$$

$$\sigma = \sqrt{6.25} = 2.5 \qquad (2.5)^2 = 6.25$$



LINEARITY OF μ.

$$\mu_{x+b} = \frac{x_1+b + x_2+b + \cdots + x_n+b}{n} = \mu_x + b$$

SO $\mu_{x+b} = \mu_x + b$

Also $\mu_{ax} = \dfrac{a x_1 + a x_2 + \cdots + a x_n}{n} = a\mu_x$

so $\mu_{ax} = a\mu_x$

Couple these $\mu_{ax+b} = \mu_{ax} + b = a\mu + b$

so $\mu_{ax+b} = a\mu_x + b$

SCALE CHANGE (on a)

LOCATION CHANGE (on b)

Example x = list of F° temps Know $C^\circ = \frac{5}{9}(F^\circ - 32)$

So if $\mu_x = 59.3^\circ F \Rightarrow \left[\mu_{\frac{5}{9}(x-32)} = \frac{5}{9}(59.3 - 32)\right]$

ALSO $\sigma^2_{x+\cancel{b}} = \left[(x_1+\cancel{b}-(\mu_x+\cancel{b}))^2+\cdots+(x_n+\cancel{b}-(\mu_x+\cancel{b}))^2\right]/n$

SO $\sigma^2_{x+\cancel{b}} = \sigma_x^2$ SO TOO $\sigma_{x+\cancel{b}} = \sigma_x$

ALSO $\sigma^2_{ax} = \left((ax_1-a\mu_x)^2+\cdots+(ax_n-a\mu_x)^2\right)/n$

$= a^2\left((x_1-\mu_x)^2+\cdots+(x_n-\mu_x)^2\right)/n$

SO $\sigma^2_{ax} = a^2\sigma_x^2 \Rightarrow \sigma_{ax} = |a|\sigma_x$

COUPLE THESE $\sigma^2_{ax+\cancel{b}} = a^2\sigma_x^2 \Rightarrow \sigma_{ax+\cancel{b}} = |a|\sigma_x$