Key 2b

4th and 5th Bulleted Exercise Details

4th Bulleted Exercise

Determine the mean and standard deviation of the number thrown by a six-sided die (equal probability on each one through six)

<table>
<thead>
<tr>
<th>Sides of a die</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.12</td>
<td>.15</td>
<td>.18</td>
<td>.20</td>
<td>.10</td>
<td>.08</td>
<td>.04</td>
</tr>
</tbody>
</table>

\[ \mu = \sum xP(x) = 0(.012) + 1(.15) + 2(.18) + 3(.20) + 4(.10) + 5(.08) + 6(.04) \]

Mean expected value = 0 + .15 + .36 + .72 + .40 + 2.00 + 1.40 = 2.45

- Answer: \( \mu = 2.45 \)

In order to find the standard deviation of the six-sided die first find the variance

\[ \sigma^2 = \sum (x - \mu)^2 P(x) = \sum x^2 P(x) - \mu^2 \]

\[ \sigma^2 = 0(.012) + 1(.15) + 4(.18) + 9(.20) + 16(.10) + 25(.08) + 36(.04) \]

\[ \sigma^2 = 8.47 - (2.45)^2 \]

\[ \sigma^2 = 2.9675 \]

\[ \sigma = \sqrt{\sigma^2} \]

- Answer: \( \sigma = 1.7244 \)

5th Bulleted Exercise

Calculate the mean and standard deviation of the number of heads thrown in 100 tosses of a coin
(and also for the number of heads thrown in 10,000 tosses at a coin). Additionally (simulation), toss a die 100 times and calculate the average of 100 numbers thrown uppermost. Does this sample average come close to the mean used calculated from the probability model?

100 tosses at a coin

\[ \mu = np \]

\[ \sigma = \sqrt{np(1-p)} \]

So: \[ \mu = 100(.50) \]

Answer: \[ \mu = 50 \]

\[ \sigma = \sqrt{100(.50)(1-.50)} \]

So: \[ \sigma \approx 7.07 \]

Answer: \[ \sigma = 5 \]

10,000 tosses at a coin

\[ \mu = 10,000(.50) \]

Answer: \[ \mu = 5,000 \]

\[ \sigma = \sqrt{10,000(.50)(.50)} \]

\[ \sigma = 50 \]

Answer: \[ \sigma = 50 \]

Simulation

I use RandInt on my calculator to obtain the average of the random digits on a six-side die rolled 100 times.

So on my calculator I typed \((1, 6, 100)\) which
represent: \( \left( 1, 0.1, 1, 0.1, 0 \right) \). So now that I had all
of my random numbers,
I wanted more
numbers
I found out the average
from
I looked at all of the 100 numbers
I figured how
the random numbers
on
the other
due

to occur. I came up with
2.3. Which was
exactly the mean
from the probability made

* Bonus problems

1. Determine: a) \( \mu \), b) \( \sigma^2 \), c) \( \sigma \), d) \( \bar{x} \),
e) \( S = \frac{1}{n-1} \sum (x_i - \bar{x})^2 \)
for \( x = 2.3, 5.4, 7.9, 6.6, 2.1, 3 \)

a) \( \bar{x} = 4.86 \) (calculator answer)
\[ \mu = \frac{2.3 + 5.4 + 7.9 + 6.6 + 2.1}{5} = 4.86 \]

b) \( \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} \)
\[ \sigma^2 = \frac{\sum (x_i - 4.86)^2}{5} \]
\[ \sigma^2 = \frac{\sum (2.3 - 4.86)^2 + (5.4 - 4.86)^2 + (7.9 - 4.86)^2 + (6.6 - 4.86)^2 + (2.1 - 4.86)^2}{5} \]
\[ \sigma^2 = \frac{(-2.56)^2 + (0.54)^2 + (3.04)^2 + (1.74)^2 + (-2.74)^2}{5} \]
\[ \sigma^2 = \frac{6.5536 + 0.2916 + 9.2416 + 3.0276 + 7.5176}{5} \]
\[ \sigma^2 = \frac{30.988}{5} \]
\[ \sigma^2 = 6.19776 \]
\( \sigma = 2.312228342 \text{ (calculator answer)} \)

\[ \sigma = \frac{\sqrt{\sum (x_i - \mu)^2}}{n} \]

\[ \sigma = \sqrt{5.3404} \]

\[ \sigma = 2.312228342 \]

\( s = \sqrt{\frac{\sum (x - \mu)^2}{n-1}} \]

\[ s = \sqrt{2.312228342} \text{ (calculator answer)} \]

\[ s = \sqrt{2.312228342} \]

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\( \text{no question} \)

\( \text{no question} \)

Refer to problem 2. For any fixed time, the total casino winnings, up to that point in time, approximate a normal (i.e., "bell") curve with

\[ \text{bell curve mean} \approx \mu \text{ and bell curve standard deviation} \approx \sigma \]
a) Evaluate the interval given
\[ 0.4417 + 0.3573 = 0.7990 \]
\[ 0.4417 - 0.3573 = 0.0844 \]

b) What is the chance the casino earns a total of less than \[ 0.4117(10,000) \]
cash $10,000 plus?
\[ \approx 0.1117 \]

\[ \text{Bell Curve} \]
\[ 36.142 \]
\[ 48.433 \]
\[ 0.1117 \]
\[ 95.00 \]

(c) What is the chance the casino earns a total of less than
\[ 0.4117(10,000) - 1.96(0.3573)(10,000) \]
after 1,000 games?
\[ \approx 0.3283 \]

\[ \text{Bell Curve} \]
\[ 32.83 \]
\[ 36.9192 \]
\[ 48.433 \]
\[ 25.00 \]