Probability, Random Variables, Expectation, Std Deviation

1. A box has colored balls: 10 R, 6 G, 4 Y. Successive draws are without replacement and with equal probability on the balls then remaining.
   
a. P(R1) (first ball is red).

b. P(G2).

c. P(G2 |\_\_ G1).

d. P(G2 |\_\_ G1^c).

e. Using the multiplication rule
   \[ P(G1 \text{ and } G2) = P(G1) \cdot P(G2 |\_\_ G1) \]

f. Using the addition rule
   \[ P(G2) = P(G1 \text{ and } G2) + P(G1^c \text{ and } G2) - 0 \text{ (no overlap)} \]

e. If we replace ball 1 before the second draw, P(G2 |\_\_ G1^c).
2. A red cube is marked with \{2, 2, 5, 8, 9, 15\}. A green cube is marked with \{4, 6, 6, 7, 11, 13\}.
   a. \(P(R > G)\).

   b. \(P(R > G \mid R = 8)\).

   c. \(P(R + G = 7)\).

3. A bent coin turns up heads with probability 0.62. The coin is tossed 12 times.
   a. The number of ways that one may arrange 4 letters H and three letters T into a line.

   b. The probability of the sequence H H H H T T T.

   c. The probability of the sequence H H H T H T T.

   d. The aggregate probability of all sequences that have four H and three T.

   e. The binomial probability of getting four heads in seven tosses.

4. Events A, B have
P(A) = 0.7   P(B) = 0.8   P(A and B) = 0.6

a. Complete a Venn diagram for the above information.

b. P(A and B^C).

c. P(A or B).

5. For the toss of a red die with faces \{2, 2, 5, 8, 9, 15\}:

   a. E R

   b. Variance R.

   c. E (total of 100 independent tosses of the die).

   d. Variance(total of 100 independent tosses of the die).

   e. Standard deviation of the total of 100 independent tosses of the die.
f. Sketch the normal approximation of the distribution of the total of 100 independent tosses of the die. Identify (c) and (e) as recognizable elements of your sketch.

6. For the probability distribution

<table>
<thead>
<tr>
<th>x</th>
<th>14</th>
<th>23</th>
<th>11</th>
</tr>
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<tbody>
<tr>
<td>p(x)</td>
<td>0.13</td>
<td>0.29</td>
<td>0.58</td>
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</tbody>
</table>

a. $E(X)$.

b. $Var(X)$.

c. $E(5X - 9)$.

d. $Var(5X - 9)$.

e. If $X_1$ and $X_2$ are independent samples from this distribution:

$E(2X - 6Y + 4)$ independence not required;

$Var(2X - 6Y + 4)$. 