Homework assignment due at the beginning of class Wednesday, 8-4-10. Read Key 52 and consult the material below and the readings of 8-2-10.

Here are class lists for sections 202-202. Your number on the ClassList will be keyed to the assignments below.

Random digits and random sampling. Random digits are statistically independent draws (i.e. equal probability with-replacement) from \{0, 1, ..., 8, 9\}. I've prepared tables of random digits for you to use.

Students listed "No. 1" on their Class List (in section 202 Barrons) are
assigned the first block of random digits, which begins 876 90... (see blocks of random digits appended below). I simply want students within each section to work with different blocks of random digits, which is why there are 18 blocks of random digits, one for each student.

Your assigned block of random digits can be used to obtain a random sample. For example, you can use random digits to sample random pages from the 209 pages of the book. Barron's random digits begin

876 904 531 090 ...

They happen to be arranged in threes. She could set up a correspondence between digit triples, all of which are equally likely from 000 through to 999:

<table>
<thead>
<tr>
<th>digit triplet</th>
<th>corresponding page</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>none</td>
</tr>
<tr>
<td>001</td>
<td>1</td>
</tr>
<tr>
<td>002</td>
<td>2</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>010</td>
<td>10</td>
</tr>
<tr>
<td>011</td>
<td>11</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>099</td>
<td>99</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>209</td>
<td>209</td>
</tr>
<tr>
<td>210</td>
<td>none</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>999</td>
<td>none</td>
</tr>
</tbody>
</table>

In order for each page to have the same chance of being selected we use a triplet 001 for page 1. Every page must face the same hurdle before being admitted to the sample, in this case the hurdle is to have its three digit correspondent come up.
1. With replacement equal probability sample.

a. Use your block of random digits to select an equal probability with replacement sample of \( n = 30 \) pages from the textbook.

b. Score each sample page (above) with \( x = \) number of times the header "KEY EXAMPLE" occurs on that page.

c. Use your sample data from (b) to determine

\[ \bar{x} \]

\[ s \]

68% z-Cl for \( \mu \)

In class, we will calculate the (grand) average of all 18 students' values for sample mean. Although it is not the actual population mean \( \mu \) it is likely to be very close. We'll see if around 68% of the classes' z-Cl cover this grand average.
2. Without replacement equal probability sample.

Set up the 2-digit correspondence 01\rightleftharpoons page 1, ..., 87\rightleftharpoons page 87, ... other pairs\rightleftharpoons none.

Starting again at the beginning of your block of random digits peruse consecutive pairs of random digits, selecting random pages from only pages 1 through 87 of the textbook. Student #1 has digits beginning
\[ 876 904 531 090 491 806 584 704 102 709 \ldots \]
Grouping into consecutive pairs
\[ 87 69 04 53 10 90 49 18 06 58 47 04 10 27 \ldots \]
see that 04 occurs twice. If we want to sample with equal probability all we have to do is skip over any duplicate. That is, if we skip all repeats of 04 the remaining unseen page numbers the unseen page numbers remain equally likely to get into the sample.

So the student above obtains equal probability without replacement sample pages from the range 01 through 87 beginning as follows:

digit pairs: \[ 87 69 04 53 10 90 49 18 06 58 47 04 10 27 \ldots \]
page selected: \[ 87 69 4 53 10 49 18 6 58 47 27 \ldots \]

a. Use your block of random digits to select an equal probability with OUT replacement sample of n = 30 pages from the textbook.
b. Score each sample page (above) with \( x = \text{number of times you see (in capital letters) "KEY EXAMPLE" on that page.} \)

c. Use your sample data from (b) to determine

\[ \bar{x} \]

\[ s \]

68% z-Cl for \( \mu \)

**Note:** In class, we will calculate the (grand) average of all 18 students' values for sample mean. Although it is not the actual population mean \( \mu \) it is likely to be very close to \( \mu \). We'll see if around 68% of the class' z-Cl cover this grand average.

3. **Achieving a given precision by choosing a large sample.** Key 52 discusses how to choose sample size \( n \) in order to ensure that a 95% z-Cl (in with replacement case) is not too wide. The form is:

\[ \bar{x} \pm 1.96 \frac{s}{\sqrt{n}} \]

Unfortunately, you don't know what \( s \) will be until you get data. What to do? Take a preliminary sample. Estimate \( \sigma \) by means of the sample standard deviation \( s_{\text{prelim}} \) of this preliminary sample. If

\[ 1.96 \frac{s_{\text{prelim}}}{\sqrt{n_{\text{prelim}}}} \leq W \]

then you are done since your regular z-Cl already has the desired narrowness specified by \( W \). Otherwise, solve for \( n_{\text{final}} \) in

\[ 1.96 \frac{s_{\text{prelim}}}{\sqrt{n}} = W \] (\( W \) being any desired half-width)
\[ n_{\text{final}} = \left( \frac{1.96 \times s_{\text{prelim}}}{W} \right)^2 \]

Then continue sampling to the larger sample size \( n_{\text{final}} \). Your 95% z-Cl is then (approximately)

\[ \bar{x}_{\text{final}} \pm W \]

which is what you wanted to achieve.

Be prepared to suffer large \( n_{\text{final}} \) if you want \( W \) to be small (precise CI).

a. An experimenter wishes to estimate the mean failure pressure (psi) for a particular type of tire. They would like a 95% z-Cl of the precision 

\[ \bar{x} \pm 10 \text{ psi} \]

A preliminary sample of 100 tires produces a sample standard deviation of \( s_{\text{prelim}} = 42 \text{ psi} \).

Determine the recommended total sample size

\[ n_{\text{final}} = \left( \frac{1.96 \times s_{\text{prelim}}}{W} \right)^2 \]

Check that the recommended \( n_{\text{final}} \) is not greater than 100. It means that the needed precision has already been achieved. Therefore, give the ordinary z-Cl from the data already in hand if it is found that \( x_{\text{prelim}} = 192.32 \text{ psi} \)

b. With the data of (a), suppose we really desire a 95% z-Cl of \( \bar{x}_{\text{final}} \pm 1 \text{ psi} \) and are perhaps willing to employ the hybrid method. Determine

\[ n_{\text{final}} = \left( \frac{1.96 \times s_{\text{prelim}}}{W} \right)^2 \]

c. Suppose that you have continued to the recommended total sample size \( n_{\text{final}} \) (you were allowed to include the initial 100 sample values) and your sample mean of all \( n_{\text{final}} \) scores is \( \bar{x}_{\text{final}} = 193.84 \). Give the hybrid z-Cl

\[ \bar{x}_{\text{final}} \pm 1 \text{ psi} \]

d. What margin of error will you quote for the hybrid method?

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**Blocks of random keyed to position in the ClassList.**

```plaintext
1
876 904 531 090 491 806 584 704 102 709 291 034 862 346 865 461 522 425 713 009 872 399 431 142 702 706 623
```