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**Assignment for 8 - 6 - 10. Homework due at the start of class.**

**Estimating the difference  $\mu_1 - \mu_2$  of the population means of two groups.**

**Case of paired comparisons.** Scores

$$(x_i, y_i) \quad i \leq n$$

are observed on  $n$  subjects selected with equal probability and with replacement from a population.

**Example A:** Subjects are sampled from a population of patients who might benefit from a new medication. Before ( $x$ ) and after ( $y$ ) measurements of platelet counts are taken for each patient. These are possible dependent measurements. That is, a given patient's count after medication may be statistically linked to their count before medication. Give a 95% z-CI for the population mean difference score  $\mu_D = \mu_{\text{before}} - \mu_{\text{after}}$ . Do this by ignoring the before and after scores and simply work with the difference scores  $d = x - y$  directly:  $\bar{d} \pm 1.96 s_d / \sqrt{n}$  where  $n$  is the number of difference scores (same as the number of subjects).

before after difference

1367	1839	-473
723	946	-224
217	274	-58
310	395	-85
39	53	-14
1362	1832	-471
210	265	-56
1	10	-9
932	1233	-302
1237	1656	-420
1059	1409	-350
37	51	-14
63	81	-19
397	509	-112
33	46	-13
543	703	-161
203	256	-54
214	270	-57
7	16	-10
37	50	-14
1231	1649	-418
1379	1856	-477
1478	1995	-518
836	1102	-266
1453	1960	-508
105	132	-28
561	728	-167
127	160	-34
1014	1347	-333
177	224	-47
84	107	-23
98	124	-27
1	10	-9
329	420	-91
276	351	-75
14	24	-10
1112	1482	-371

Case of independent samples

independent samples  $x_i$   $i \leq n_x$  from population of  $x$

independent samples  $y_i$   $i \leq n_y$  from population of  $y$

with  $x$  samples all independent of  $y$  samples.

**Example B.** Related to example A above except that we sample a number of subjects to receive a

placebo and, independently of these, another sample of subjects to receive the medication. If the population of potential subjects is large compared with the numbers sampled then approximate independence is achieved.

placebo {1460, 117, 419, 364, 555, 66, 207, 575, 698, 94, 648, 669, 261, 676, 200,  
1157, 8, 282, 17, 1069, 609, 0, 0, 1548, 700, 859, 398, 410, 8, 447, 31, 1314}

med {164, 136, 842, 339, 94, 726, 91, 768, 466, 26, 1943, 78, 920, 1886, 10, 569, 1404,  
1880, 751, 214, 35, 102, 64, 10, 537, 23, 479, 664, 944, 834, 17, 189, 1930, 20, 1260, 13}

It is natural to estimate the difference  $\mu_x - \mu_y$  of population means by the difference  $\bar{x} - \bar{y}$  of sample means.

The standard deviation of  $\bar{x} - \bar{y}$  is  $\sqrt{\sigma_x^2/n_x + \sigma_y^2/n_y}$  because  $\bar{x}$  independent of  $\bar{y}$  implies  $\text{Var}(\bar{x} - \bar{y}) = \text{Var}(\bar{x}) \oplus \text{Var}(\bar{y})$ . So a 95% z-CI is obtained by **plugging in estimates** in place of the unknown population sigmas:

$$\bar{x} - \bar{y} \pm 1.96 \sqrt{s_x^2/n_x + s_y^2/n_y}$$

Homework due at the start of class 8-6-10.

1. Solve for the 95% z-CI for the difference  $\mu_x - \mu_y$  in the paired comparison of example A.
2. Solve for the 95% z-CI for the difference  $\mu_x - \mu_y$  in the independent samples setup of example B.
3. A table of data blocks A is posted separately for this assignment. Repeat (1) for your data block

A corresponding to your No. on the class list.

4. A table of data blocks B is posted separately for this assignment. Repeat (2) for your data block B corresponding to your No. on the class list.