Due a start of class 8-2-10.

1. Let $X$ be the number of tosses to obtain the first head.
   
   a. Guess $\mu = E(X)$ (it is intuitive).
   
   \[ \mu = \frac{1}{2}(1) + \frac{1}{2}(1+\mu) \]
   
   \[ \Rightarrow \mu = 2 \]
   
   b. Can you guess $\sigma$?
   
   \[ \Rightarrow \mu = 2 \]
   
   c. Let $x_1$ denote the number of tosses you have to make to get the first head. Repeat the experiment to get $x_2$ (the number of tosses you have to make to get the first head the second time you try the experiment). Do this 30 times getting $x_1, \ldots, x_{30}$. Record the results (number of tosses required for each of 30 replications of "tossing until the first head."

   \[ 4, 3, 1, 6, 1, 1, 2, 1, 1, 1, 3, 1, 1, 2, 3, 4, 1, 4, 1, 2, 1, 1, 1, 3, 4, \]
   
   \[ 3, 4, 2, 5 \]
   
   \[ n = 30 \]
   
   \[ \bar{x} = \frac{69}{30} = 2.3 \]

   d. From your sample of $n = 30$ give $\bar{x}$ (sample mean), an estimate of $\mu$.

   \[ \bar{x} \]

   \[ s, \text{ your estimate of } \sigma \]

   \[ \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} \]

   \[ = \sqrt{\frac{29}{29}} + \frac{(5-2.3)^2}{29} + \frac{(6-2.3)^2}{29} \]

   \[ = \sqrt{1.9177} = 1.385 \]

   \[ \frac{s}{\sqrt{n}}, \text{ your estimate of the standard deviation of } \bar{x} \]

   \[ \frac{1.385}{\sqrt{30}} = .253 \]

   MOE (margin of error for $\bar{x}$) = 1.96 $\frac{s}{\sqrt{n}}$

   \[ 1.96 \left( \frac{1.385}{\sqrt{30}} \right) = .496 \]
68%
-95% z-based CI for \( \mu \)
\[
+ 2.3 + .253 = 2.553
\]
\[
- 2.3 - .253 = 2.047
\]

If \( \mu \) is not in your interval then a "bad" event has occurred.
What is the probability of this "bad" event?

In class, 7/13 did not cover: = .54 w/ 68%, 32% would not cover

Around what fraction of the class should have an 80% t-Cl containing \( \mu \)?

Prepare a histogram of your 30 numbers, does it look at all as though \( X \) is normal distributed?

\[
\text{no.}
\]

2. Let \( X \) = the number of heads in 10 tosses of a coin. Although \( X \) is not normally distributed (it is binomial) the distribution is not far from normal with mean np, and standard deviation \( \sqrt{np(1-p)} \).
For \( n = 3 \) times toss a coin 10 times recording the number of heads \( x_1, x_2, x_3 \) in each of the three experiments.

From your sample of \( n = 3 \) give

\[
x \text{ (sample mean), an estimate of } \mu
\]
\[
\bar{x} = \frac{15}{3} = 5
\]

\[
s, \text{ your estimate of } \sigma
\]
\[
\sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{(4 - 5)^2}{2} + \frac{(5 - 5)^2}{2} + \frac{(6 - 5)^2}{2}}
\]
\[
= \sqrt{\frac{1}{2} + 0 + \frac{1}{2}} = \sqrt{1} = 1
\]
\[
\frac{\sigma}{\sqrt{n}}, \text{ your estimate of the standard deviation of } \bar{x} \\
\frac{1}{12} = .707
\]

\[df = 3 - 1 = 2\]

\[t - \text{MOE (margin of error for } \bar{x}) = t_{0.025} \frac{\sigma}{\sqrt{n}} \]

\[4.303(.707) = 3.04\]

80\% t-based CI for \(\mu\)

\[5 - (1.886)(1) = 3.114 \quad 5 + (1.886)(1) = 6.886\]

If \(\mu\) is not in your interval then a "bad" event has occurred.
What is the probability of this "bad" event? \(\mu = 5\)

W/ 80\% CI, 20\% would not cover

Around what fraction of the class should have an 80\% t-Cl containing \(\mu\)?

3. A 95\% z-Cl for \(\mu\) based on a large sample selected with replacement from a population is given as [3.884, 3.9170].

\[
\text{MOE} = \frac{1}{2} \text{ diff of upper + lower CI} \\
= 3.9170 - 3.884 = 0.033/2 = 0.0165
\]

Interval for 68\% confidence

\[
x = \frac{3.884 + 3.9170}{2} = 3.9005
\]

95\% z-Cl if instead the sampling is without replacement, population size \(N = 1000\) and sample size \(n = 100\).

\[
x \pm 1.96 \sqrt{\frac{N-n}{N-1} \frac{4}{1000-100}} \]

\[
x = 3.9005 \pm 0.165 (9.499/6)
\]

\[
3.9005 \pm 0.165 (9.499/6)
\]
## Problem

### Class Data #1

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<tr>
<th>Name</th>
<th>m</th>
<th>$\bar{x}$</th>
<th>$\Delta$</th>
<th>$\frac{\Delta}{\text{in}}$</th>
<th>$1.96 \times \frac{\Delta}{\text{in}}$</th>
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<th>$\frac{95%}{\text{of} \bar{x}} - \frac{2 \times \Delta}{\text{in}}$</th>
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\( \text{9/13 Cover} \)