1. A random variable $X$ has the following probability distribution:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(x)$</td>
<td>0.4</td>
<td>0.6</td>
<td>0.2</td>
</tr>
</tbody>
</table>

a. $E X$

$$0 + 0.6 + 1 = 1.6$$

b. $E X^2$

$$0 + 0.6 + 5.2 = 5.6$$

c. Var $X$

$$\sqrt{5.6 - (1.6)^2} = \sqrt{5.6 - 2.56} = 3.04$$

d. Standard deviation of $X$

$$\sqrt{3.04} = 1.7436$$

2. A random variable $X$ has $E X = 8$, variance $X = 4$. Denote by $T$ the random total of 900 independent plays of $X$.

a. $E T$

$$8 \cdot 900 = 7200$$

b. $Var T$

$$4 \cdot 900 = 3600$$

c. Standard deviation of $T$

$$\sqrt{3600} = 60$$
d. Sketch the approximate distribution of total $T$, clearly indicating the means and standard deviation of $T$ as recognizable elements in your sketch.

\[ Z_{three} = \frac{7360 - 7200}{60} = \frac{160}{60} = 2.6667 \]

Given $A^c :: 0.4962$

\[ 3. \text{ Events } A, B \text{ have} \]
\[ P(A) = 0.2 \]
\[ P(B \mid A) = 0.6 \]
\[ P(B \mid A^c) = 0.3 \]

a. $P(A \text{ and } B)$
\[ 0.2 \cdot 0.6 = 0.12 \]

b. $P(A^c \text{ and } B)$
\[ 0.8 \cdot 0.3 = 0.24 \]

c. $P(B)$
\[ 0.2 \cdot 0.6 + 0.8 \cdot 0.3 = 0.12 + 0.24 = 0.36 \]

d. $P(A \mid B)$
\[ \frac{P(A \text{ and } B)}{P(B)} = \frac{0.12}{0.36} = 0.3333 \]
4. In terms of constants a, b, c and Var X, Var Y of independent random variables X, Y,
   a. $E(aX - bY + c) = aE(X) - bE(Y) + c$
   b. $Var(aX - bY + c) = a^2Var(X) + b^2Var(Y)$

5. Box 1 has 4R 4G
   Box 2 has 3R 9G
   Box 1 will be chosen with probability 0.7
   Box 2 will be chosen with probability 0.3

A ball will be selected with equal probability from the box chosen.
   a. Intuitively, is $P(\text{Box 1} | R)$ or $P(\text{Box 1})$ the larger? Why?
      $P(\text{Box 1}) = 0.7$
      So, $P(\text{Box 1} | R) > P(\text{Box 1})$
      $P(\text{Box 1} | R) = 0.8235$, is larger.
   b. $P(\text{Box 1 and R})$
      $0.7 \cdot \frac{4}{8} = \frac{28}{15} = 0.35$
   c. $P(R)$
      $P(\text{Box 1} | R) = 0.35 + 0.075 = 0.425$
   d. $P(\text{Box 1} | R)$
      $\frac{P(\text{Box 1} \text{ and R})}{P(R)} = \frac{0.35}{0.425} = 0.8235$
6. Calculator may be used.

\[
\begin{array}{cccccc}
    x & (x - \text{Mean}[x])^2 & x^2 \\
    2 & 324 & 4 \\
    4 & 64 & 256 \\
    4 & 64 & 256 \\
    8 & 144 & 512 \\
    10 & 484 & 1000 \\
    - & - & - \\
    28 & 216 & 200 \\
\end{array}
\]

(totals at bottom)

a. Standard deviation \( \sigma \) of list \( x \).

\[
\frac{1732}{5} = 346.4
\]

\[ \sigma = 2.9394 \]

b. Median of list \( x \).

\[ 4 \]

c. Standard deviation \( \sigma \) of list \(-5x + 8\).

\[
5 \cdot 2.9394 = 14.697
\]

d. Median of list \(-5x + 8\).

\[-12\]

e. Height of the probability histogram for list \( x \) over the interval \([3, 9]\).

\[
\frac{\frac{1}{5}}{9 - 3} = \frac{\frac{1}{5}}{6} = 0.1
\]

f. Sketch the cumulative probability distribution for list \( x \). Be sure it rises between 0 and 1 with jumps of appropriate amounts.
7. Poisson. We expect on average $21.6$ falcon sightings per week and the number of sightings is thought to follow a Poisson distribution. Recall that for the Poisson the mean and variance are the same.

a. Sketch the approximate distribution of the Poisson for this case. Label the numerical mean and standard deviation as recognizable numerical elements of your sketch.

b. Give a $68\%$ interval for the number of sightings in a given week.

c. The formula for Poisson $p(x)$ is $e^{-\lambda} \frac{\lambda^x}{x!}$ for $x = 0, 1, \ldots$

$$p(20) = e^{-21.6} \frac{(21.6)^{20}}{20!}$$

(first write all appropriate numbers in the formula, evaluate the factorial, then evaluate using calculator).

8. Binomial. Each time a casting from production is x-rayed there is probability $0.1$ it will be found defective. These events are thought to be pretty much independent.

a. The probability that the first six castings x-rayed are:

<table>
<thead>
<tr>
<th>def</th>
<th>not-def</th>
<th>def</th>
<th>not-def</th>
<th>def</th>
<th>not-def</th>
<th>non-def</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

$$= (0.1)^4 (0.9)^2 = 0.000081$$

b. The number of ways to select four of six positions as the only ones defective.

$$\binom{6}{4} = \binom{6}{2} = 15$$

c. The binomial probability that out of six castings there are precisely four that are found defective when x-rayed.

$$15 \cdot (0.1)^4 (0.9)^2 = 0.0012$$

d. Sketch the approximate distribution of the number $X$ of castings, out of $100$ castings x-rayed, that are found defective. Identify the mean and standard deviation of $X$ numerically in your sketch.