Final exam prep 8 - 13 - 10 Try these. Not handed in. Will go over in class. Report errors.

1. **z-Cl for p.** An equal probability with replacement sample of 100 persons is selected from customers of a store by randomly alerting a clerk at checkout. Customers are offered a choice of one of two items A or B. It is found that 62 out of 100 choose item A over item B. The form of a 95% z-Cl for the population fraction $p$ of all customers who would choose item A over item B is

$$\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

a. Evaluate the CI for the data given.

b. From the formula above identify

point estimate of $p$ and its value for this data

MOE for $\hat{p}$ and its value for this data

point estimate of sd of $\hat{p}$ and its value for this data

c. $P(p$ is covered by $\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}) \sim$

d. Percentage of users of such CI whose CI covers $p$ ~ (assumes the users operate independently)
2. **z-CI for \( \mu \) with or without replacement.** An equal probability with replacement sample of 40 pages is selected from a textbook of 469 pages. Each sample page is scrutinized to discover the number of errors "x" (what this means is carefully codified). It is found that these 40 x-scores have sample mean 0.73 and sample standard deviation s = 1.51.

a. Give mathematical form of a 95% z-CI for \( \mu \), the average number of errors per page in the entire book.

b. From the formula above identify

point estimate of \( \mu \) and its value for this data

MOE for \( \bar{X} \) and its value for this data

point estimate of sd of \( \bar{X} \) and its value for this data

c. From rule or z-table (or your calculator) determine z for

68% CI

83% CI

d. How is the 95% z-CI to be modified if it is learned that the data was actually obtained from a without-replacement equal probability random sample? Write the explicit form of the CI and evaluate it.
3. **Hybrid z-CI to achieve 95% z-Cl of form** $\bar{x}_{\text{final}} \pm 0.2$. In #2a, regard the sample of 40 as a preliminary with-replacement equal probability sample of $n_{\text{preliminary}} = 40$ (preliminary sample mean 0.73 and preliminary sample standard deviation $s_{\text{preliminary}} = 1.51$). We desire a 95% hybrid z-Cl for $\mu$ of the form $\bar{x}_{\text{final}} \pm 0.2$.

   a. Evaluate the MOE for the preliminary 95% z-Cl (same as in 2a). Does it already have at least the precision 0.2?

   b. Equating $(1.96 \frac{s_{\text{preliminary}}}{\sqrt{n_{\text{final}}}} = 0.2$ solve for the final sample size $n_{\text{final}}$ needed by the hybrid z-Cl. Is $n_{\text{final}} > 40$? If so, your answer to (a) must have been NO, you did not have the needed precision at $n = 40$.

   c. Suppose we continue sampling up to $n_{\text{final}}$ and find that $\bar{x}_{\text{final}} = 0.77$. Give the 95% hybrid z-Cl for $\mu$. 
4. Match each of the CI below to their intended use at left (one "CI" is never used).

<table>
<thead>
<tr>
<th>Hybrid z-Cl</th>
<th>$d \pm z \frac{s_d}{\sqrt{n}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>For all $n &gt; 1$, normal population only</td>
<td>$\hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$</td>
</tr>
<tr>
<td>For $\mu_x$, large $n$, with-replacement sample</td>
<td>$x \pm z \frac{s}{\sqrt{n}}$ FPC</td>
</tr>
<tr>
<td>For $\mu$ without repl sample</td>
<td>$x \pm t_{\alpha, df} \frac{s}{\sqrt{n}}$ FPC</td>
</tr>
<tr>
<td>Difference of means, unpaired, independent data</td>
<td>$x_{\text{final}} \pm W$</td>
</tr>
<tr>
<td>Difference of means, paired data</td>
<td>$(\bar{x} - \bar{y}) \pm z \sqrt{s^2_x / n_x + s^2_y / n_y}$</td>
</tr>
<tr>
<td>For population proportion</td>
<td>$x \pm t_{\alpha, df} \frac{s}{\sqrt{n}}$</td>
</tr>
</tbody>
</table>
5. **A sample of n = 5 from a normal population.** Suppose the sample mean is 3.79 and the sample sd is s = 2.45. Determine

a. MOE

b. 95% CI for μ

c. In this setup, with samples from a normal population distribution, the hybrid method works for any small preliminary sample with n > 1. Using the appropriate replacement of 1.96 find the n required for a hybrid CI to achieve precision

\[ \bar{x}_{\text{final}} \pm 0.2 \]

Keep in mind, our preliminary sample of only n = 5 works because the population distribution is normal and we are using the correct replacement for 1.96.

d. If you do continue to the recommended sample size and find that \( \bar{x}_{\text{final}} = 3.65 \) what is the hybrid 95% CI?

e. Refer to (d). What is the MOE?