1. **z-Cl for p.** An equal probability with replacement sample of 100 persons is selected from customers of a store by randomly alerting a clerk at checkout. Customers are offered a choice of one of two items A or B. It is found that 62 out of 100 choose item A over item B. The form of a 95% z-Cl for the population fraction \( p \) of all customers who would choose item A over item B is

\[
\hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]

a. Evaluate the CI for the data given.

\( \hat{p} = \frac{62}{100} = 0.62 \)

CI for \( p \) is \( 0.62 \pm 1.96 \sqrt{\frac{0.62 \times 0.38}{100}} \)

b. From the formula above identify

point estimate of \( p \) and its value for this data

0.62 (estimate around 62% of customers favor A)

MOE for \( \hat{p} \) and its value for this data

\[ 1.96 \sqrt{\frac{0.62 \times 0.38}{100}} \]

point estimate of sd of \( \hat{p} \) and its value for this data

\[ \sqrt{\frac{0.62 \times 0.38}{100}} \]

c. \( P(\hat{p} \text{ is covered by } \hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}) \approx 0.95 \)

d. Percentage of users of such CI whose CI covers \( p \approx 95\% \)
(assumes the users operate independently)
2. **z-Cl for \( \mu \) with or without replacement.** An equal probability with replacement sample of 40 pages is selected from a textbook of 469 pages. Each sample page is scrutinized to discover the number of errors "x" (what this means is carefully codified). It is found that these 40 x-scores have sample mean 0.73 and sample standard deviation \( s = 1.51 \).

a. Give mathematical form of a 95% z-Cl for \( \mu \), the average number of errors per page in the entire book.

\[
\bar{x} \pm z \frac{s}{\sqrt{n}} = 0.73 \pm 1.96 \frac{1.51}{\sqrt{40}}
\]

b. From the formula above identify

- point estimate of \( \mu \) and its value for this data
  \( \bar{x} = 0.73 \)
- MOE for \( \bar{x} \) and its value for this data
  \[
  1.96 \frac{s}{\sqrt{n}} = 1.96 \frac{1.51}{\sqrt{40}}
  \]
- point estimate of sd of \( \bar{x} \) and its value for this data
  \[
  \frac{s}{\sqrt{n}} = \frac{1.51}{\sqrt{40}}
  \]

c. From rule or z-table (or your calculator) determine z for

- 68% CI 1.00 rule of thumb
- 83% CI z-table entry for z = 0.83/2 = 0.415 \( \sim 1.3722 \)

d. How is the 95% z-Cl to be modified if it is learned that the data was actually obtained from a without-replacement equal probability random sample? Write the explicit form of the CI and evaluate it.

\[
\text{With FPC} = \sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{469-40}{469-1}} \text{ so } \bar{x} \pm z \frac{s}{\sqrt{n}} \text{ FPC} = 0.73 \pm 1.96 \frac{1.51}{\sqrt{40}} \sqrt{\frac{469-40}{469-1}}
\]
3. Hybrid z-Cl to achieve 95% z-Cl of form $\bar{x}_{final} \pm 0.2$. In #2a, regard the sample of 40 as a preliminary with-replacement equal probability sample of $n_{\text{preliminary}} = 40$ (preliminary sample mean 0.73 and preliminary sample standard deviation $s_{\text{preliminary}} = 1.51$). We desire a 95% hybrid z-Cl for $\mu$ of the form

$\bar{x}_{final} \pm 0.2$.

a. Evaluate the MOE for the preliminary 95% z-Cl (same as in 2a). Does it already have at least the precision 0.2?

\[
1.96 \frac{s}{\sqrt{n}} = 1.96 \frac{1.51}{\sqrt{40}} \approx 0.467954 > 0.2
\]

(not meeting the desired precision)

b. Equating \(1.96 \frac{s_{\text{preliminary}}}{\sqrt{n_{\text{final}}}} = 0.2\) solve for the final sample size $n_{\text{final}}$ needed by the hybrid z-Cl. Is $n_{\text{final}} > 40$? If so, your answer to (a) must have been NO, you did not have the needed precision at $n = 40$.

\[
n_{\text{final}} = (1.96 \frac{s_{\text{preliminary}}}{0.2})^2 = (1.96 \frac{1.51}{0.2})^2 \approx 219
\]

c. Suppose we continue sampling up to $n_{\text{final}}$ and find that $\bar{x}_{\text{final}} = 0.77$. Give the 95% hybrid z-Cl for $\mu$.

$\approx 0.77 \pm 0.2$ as desired
4. Match each of the CI below to their intended use at left (one "CI" is never used).

5 Hybrid z-Cl

\[ d \pm z \frac{s_d}{\sqrt{n}} \]  

7 For all \( n > 1 \), normal population only

\[ \hat{\rho} \pm 1.96 \sqrt{\frac{\hat{\rho}(1-\hat{\rho})}{n}} \]  

For \( \mu_x \), large \( n \), with-replacement sample

\[ \bar{x} \pm z \frac{s}{\sqrt{n}} \text{ FPC} \]  

3 For \( \mu \) without repl sample

\[ \bar{x} \pm t_{\alpha, \text{df}} \frac{s}{\sqrt{n}} \text{ FPC} \]  

6 Difference of means, unpaired, independent data

\[ \bar{x}_{\text{final}} \pm W \]  

1 Difference of means, paired data

\[ (\bar{x} - \bar{y}) \pm z \sqrt{\frac{s_x^2}{n_x} \oplus \frac{s_y^2}{n_y}} \]  

2 For population proportion

\[ \bar{x} \pm t_{\alpha, \text{df}} \frac{s}{\sqrt{n}} \]
5. **A sample of n = 5 from a normal population.** Suppose the sample mean is 3.79 and the sample sd is s = 2.45. Determine

a. MOE

\[
 t_{\alpha, df} \frac{s}{\sqrt{n}} = t_{0.025, 5-1} \frac{2.45}{\sqrt{5}} = \frac{2.777}{2.45 \sqrt{5}}
\]

b. 95% CI for \( \mu \)

\[
 \bar{x} \pm t_{\alpha, df} \frac{s}{\sqrt{n}} = 3.79 \pm 2.777 \frac{2.45}{\sqrt{5}}
\]

c. In this setup, with samples from a **normal population** distribution, the hybrid method works for **any** preliminary sample with \( n > 1 \). Using the appropriate replacement of 1.96 find the \( n \) required for a hybrid CI to achieve precision

\[
 x_{\text{final}} \pm 0.2
\]

Keep in mind, our preliminary sample of only \( n = 5 \) works because the population distribution is normal and we are using the correct replacement for 1.96.

\[
 n_{\text{final}} = (2.777 \frac{s_{\text{preliminary}}}{0.2})^2 = (2.777 \frac{2.45}{0.2})^2 \sim 1158 \text{ (round up)}
\]

d. If you do continue to the recommended sample size and find that \( x_{\text{final}} = 3.65 \) what is the hybrid 95% CI?

\[
 \sim 3.65 \pm 0.2
\]

e. Refer to (d). What is the MOE?

\[
 \sim 0.2 \text{ as desired (but it sure costs a lot of sampling)}
\]