Basic large sample CI for the population mean.
Samples with-replacement, sample size n “large enough.”

$$P\left(\mu \in \boxed{\bar{X} \pm z \frac{s}{\sqrt{n}}}\right) \to P(Z \in [-z, z]) \quad \Leftarrow \text{claim}$$

as $n \to \infty$, for each $z \in \mathbb{R}$. For example,

$$P\left(\mu \in \boxed{\bar{X} \pm 1.96 \frac{s}{\sqrt{n}}}\right) \approx P(Z \in [-1.96, 1.96])$$

z	.06		
1.9	0.475	=	.95 / 2

⇐ z table

In around 95 % of samples of large n the population mean μ will be enclosed within the 95 % confidence interval

$$\left[\bar{X} - 1.96 \frac{s}{\sqrt{n}},\ \bar{X} + 1.96 \frac{s}{\sqrt{n}}\right]. \quad \Leftarrow \text{CI}$$

FPC (finite population correction) for sampling without replacement

$$P\left(\mu \in \boxed{\bar{X} \pm z \frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}}\right) \approx P(Z \in [-z, z])$$

as $n \to \infty$, $N - n \to \infty$, for each $z \in \mathbb{R}$. For example,

$n = 600$ samples without replacement

population of $N = 7000$

$$\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{7000-600}{7000-1}} = 0.956251 \Leftarrow \text{FPC}$$

vitrually no change from sampling with - replacement.

Student-t method

For sample from a NORMAL population (in control) valid for each n = 2, 3, …

DF (deg freedom) = n - 1 when using t-table

$$P\left(\mu \in \boxed{\bar{x} \pm t \frac{s}{\sqrt{n}}}\right) = P(T \in [-t, t]) \text{ for every } n > 1,$$

for each $t \in \mathbb{R}$. FOR POPULATION IN CONTROL (NORMAL).

DF	0.025
5 − 1 = 4	2.776
∞	1.96
CI	95 %

← t-table

So a 95 % CI based upon a sample of n = 5 from a NORMAL population would be $\boxed{\bar{x} \pm 2.776 \frac{s}{\sqrt{5}}}$.

CI for a population proportion p with vs without replacement large n (resp. n, N-n)

$$P\left(p \in \boxed{\hat{p} \pm z \frac{\sqrt{\hat{p}\,\hat{q}}}{\sqrt{n}}}\right) \approx P\,(Z \in [-z,\ z])\ \text{with repl}$$

$$P\left(p \in \boxed{\hat{p} \pm z \frac{\sqrt{\hat{p}\,\hat{q}}}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}}\right) \approx P\,(Z \in [-z,\ z])\ \text{w/o repl}$$

CI for the difference of two means (independent samples of n_1 resp n_2 both large)

$$P\left(\mu_1 - \mu_2 \in \boxed{\bar{X}_1 - \bar{X}_2 \pm z\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}\right) \approx P(Z \in [-z, z])$$

for independent samples of repsective sizes n_1, n_2 respectively.

CI for the difference of two population proportions (independent samples of n_1 resp n_2)

$$P\left(p_1 - p_2 \in \boxed{\hat{p}_1 - \hat{p}_2 \pm z\sqrt{\frac{\hat{p}_1\,\hat{q}_1}{n_1} + \frac{\hat{p}_2\,\hat{q}_2}{n_2}}}\right) \approx P\,(Z \in [-z,\ z])$$

for independent samples of repsective sizes n_1, n_2 respectively.