

Confidence intervals are rather straightforward. You don't know the population μ or σ so you estimate them by \bar{x} and s respectively and the CI is

$$[\bar{x}] \pm (z \text{ or } t) [S_x / \mathbf{root(n)}].$$

Likewise, you don't know the population p so you estimate it by \hat{p} and the CI is

$$[\hat{p}] \pm z [\mathbf{root(p\hat{HAT} q\hat{HAT})} / \mathbf{root(n)}].$$

Similarly, for paired data

$$[\bar{D}] \pm (z \text{ or } t) [S_d / \mathbf{root(n)}].$$

while for independent samples

$$[\bar{x}_1 - \bar{x}_2] \pm z [\mathbf{root(S_1^2 / n_1 + S_2^2 / n_2)}]$$

and

$$[\hat{p}_1 - \hat{p}_2] \pm z [\mathbf{root(p_1\hat{HAT} q_1\hat{HAT} / n_1 + p_2\hat{HAT} q_2\hat{HAT} / n_2)}].$$

In every case the bold expression is our estimator of the sd of our estimator, e.g. $[S_x / \mathbf{root(n)}]$ is our estimator of the sd of our estimator $[\bar{x}]$. We know that t is not to be used unless the population distribution of X or D is at least close to normal.

Tests can be based on CI. For example, if we wish to test $H_0: \mu = 16$ ounces versus $H_1: \mu$ is not 16 ounces at level $\alpha = 0.05$ we could simply create a 95% CI for μ

$$[\bar{x}] \pm (z \text{ or } t) [S_x / \mathbf{root(n)}]$$

and reject H_0 if this CI fails to cover 16 ounces. If, truly, $\mu = 16$ ounces then the chance we will (falsely) reject H_0 is 5% (after all, we are using a 95% CI). We do not approach testing in this way, by linking it to CI, because there are many exceptions to doing so.

For example, if the boundary point is p_0 then the usual test statistic is not A: $(\hat{p} - p_0) / [\mathbf{root(p\hat{HAT} q\hat{HAT})} / \mathbf{root(n)}]$. Instead, we use B: $(\hat{p} - p_0) / [\mathbf{root(p_0 q_0)} / \mathbf{root(n)}]$. The reasons for this are touched upon in your textbook so I will not repeat them here except to say that form B is widely adopted and (the main point)

more accurately achieves the specified alpha level. In actual practice, however, there is frequently no great difference between the two tests. As I work through the odd examples from chapter 8 I will do some both ways to show you this.

For testing differences, similar considerations arise in a more subtle fashion. By way of example, for testing $H_0: p_1 - p_2 = D$, you will see on pg. 347 (expression 8—13) the z-test statistic

$$z = \frac{[(\hat{p}_1 - \hat{p}_2) - D]}{[\sqrt{(\hat{p}_1 \hat{q}_1 / n_1 + \hat{p}_2 \hat{q}_2 / n_2)]}$$

This is straight from the CI (given at the outset, above)

$$[\hat{p}_1 - \hat{p}_2] \pm z [\sqrt{(\hat{p}_1 \hat{q}_1 / n_1 + \hat{p}_2 \hat{q}_2 / n_2)]}$$

But, when $D = 0$ a strange thing happens. We find on pg.346 (expression 8-11) that, for the special case $D = 0$, the preferred test statistic pools the data to form $\hat{p}_{pooled} = (x_1 + x_2) / (n_1 + n_2)$ and

$$z = \frac{[(\hat{p}_1 - \hat{p}_2) - 0]}{[\sqrt{(\hat{p}_{pooled} \hat{q}_{pooled} / n_1 + \hat{p}_{pooled} \hat{q}_{pooled} / n_2)]}$$

which is more simply expressed

$$z = \frac{[(\hat{p}_1 - \hat{p}_2) - 0]}{[\sqrt{(\hat{p}_{pooled} \hat{q}_{pooled})} \sqrt{(1/n_1 + 1/n_2)]}$$

In this course, you will be entitled to dispense with pooling, and just use the form below even for the case $D = 0$

$$z = \frac{[(\hat{p}_1 - \hat{p}_2) - D]}{[\sqrt{(\hat{p}_1 \hat{q}_1 / n_1 + \hat{p}_2 \hat{q}_2 / n_2)]}$$

Pooling only arises in the case $D = 0$ (not for $D = 10^{-100}$??? !!!).

The following rule will be enforced on exams. You may use the pooled form for the case $D = 0$ (the solutions given in the book assume that you pool). If you choose not to pool you must write “I choose not to pool” with your answer. Also, dispense with pooling on pg. 343 and exercises 18-28 altogether.