In-class 2.

1. Toss a coin 25 times. Score each toss
   x = 0 for tail, x=1 for head.

a. Record your tosses here: 0,1,1,0,1,0,0,1,1,1,1,0,1,1,1,0,0,1,0

b. Determine pHAT = #heads/ 25. Note this is xBAR.

$\bar{x} = \frac{12}{25}$

c. Estimate the standard deviation of pHAT by
   est of sd of pHAT = root(pHAT (1-pHAT))/root(n)

$= \frac{\sqrt{12/25\,(1-12/25)}}{\sqrt{25}} = .0999$

d. Use (b)(c) to determine a 95% confidence interval (1.96)
for p = probability a coin turns up heads when tossed.

$\frac{12}{25} \pm 1.96\,(.0999)$

$.48 \pm 1.96\,(.0999)$

e. If each of 50 people independently prepares a 68% confidence interval for p, around how many of these intervals will contain p = 0.5? Has your interval done so?

$\hat{p} \pm 1\,(.0999)$

$.48 \pm 1(.0999)$

f. What fraction of your classmates has their CI cover p = 0.5? Is it close to 68%?

About 20 students which is roughly 68% of the class

2. Here is a specimen population consisting of two strata.

stratum 1 scores	0	0	0	0	0	0	4	4
stratum 2 scores	0	0	8	8				

a. $N_1 =$ 8

$N_2 =$ 4

N = 8+4=12

b. $\mu_1 = \frac{4+4}{8} = 1$

$\mu_2 = \frac{8+8}{4} = 4$

$\mu = \frac{4+4+8+8}{12} = 2$

c. $\sigma_1 = \sqrt{\frac{4^2+4^2}{8} - \left(\frac{4+4}{8}\right)^2}$

$\sigma_2 = \sqrt{\frac{8^2+8^2}{4} - \left(\frac{8+8}{4}\right)^2}$

$= \sqrt{2.64/4 - 4^2}$

d. Proportional sampling of n = 6 requires how many samples from each of stratum 1 and stratum respectively?

$n_1 = n\left(\frac{N_1}{N}\right) = 6\left(\frac{8}{12}\right) = 4$

$n_2 = n\left(\frac{N_2}{N}\right) = 6\left(\frac{4}{12}\right) = 2$

e. Suppose a proportionally stratified random sample of n = 6 consists of the numbers 0 0 from stratum 1 and the numbers 0 0 0 8 from stratum 2. 0 004 00 from Stratum 2

$\bar{x} = \frac{0+0+0+4+0+0}{6} = \frac{4}{6}$

$\bar{x}_1 = \frac{0+0+0+4}{4} = 1$

$\bar{x}_2 = \frac{0+0}{2} = 0$

$s_1 = \sqrt{\frac{n_1}{n_1-1}}\sqrt{\overline{x_1^2} - (\bar{x})^2} = \sqrt{\frac{4}{4-1}}\sqrt{\frac{4^2}{4} - 1^2} = \sqrt{\frac{4}{3}}\sqrt{3} = 2$

$s_2 = \sqrt{\frac{2}{2-1}}\sqrt{0-0^2} = 0$

f. Determine the estimated s.d. of xBAR

$\frac{\sqrt{\sum_{i=1}^{2} \frac{N_i}{N} s_i^2 \frac{N_i - n_i}{N_i - 1}}}{\sqrt{n}} = \frac{\sqrt{8/12\,(4/3)(4-1)\left(\frac{8-4}{8-1}\right) + 4/12\,(0)}}{\sqrt{6}}$

$s_1^2 = \frac{n}{n-1}\left(\overline{x_1^2} - \bar{x}^2\right) = \frac{4}{3}\left(16/4 - 1^2\right)$

$s_2^2 = \frac{n_2}{n_2-1}\left(\overline{x_2^2} - \bar{x}_2^2\right) = \frac{2}{2-1}(0) = 0$

g. In terms of $\bar{x}$ and (f) determine a 95% confidence interval for μ. How will its width likely compare with a 95% CI based on a without replacement sample of n = 6 from the same population?

$\bar{x} \pm 1.96\,(f)$

It would be narrower

THIS NEXT PART IS NOT AN EXERCISE.

Example of how to randomly sample without replacement. Pick a starting block in the table of random digits. For example, start in block 31. The digits in block row 31 and column 1 are: 24130.

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I WILL SELECT 2 FROM STRATUM 2 FIRST, THEN SELECT 4 FROM STRATUM 1.

If we adopt the convention of using the first distinct (unequal) random digits from 1 to 4 we encounter to select two of the 4 numbers 0 0 8 8 from stratum 2 (without replacement) these randomly selected scores from stratum 2 would be **0 8 0 8** since these are in the 2nd and 4th positions of 0 0 8 8.

* *

Any random digit not equal to 1, 2, 3, or 4 would be skipped. Any repeated random digits would be skipped because we want samples **without replacement**.

Having otained a sample of $n_2 = 2$ from stratum 2 we then proceed to do the same with stratum 1 in order to obtain a sample of $n_1 = 4$. We begin afresh with the subsequent random digits from the table:

1 3 0 4 8 3 6 0 ...
* * * *

Using them to likewise select four from stratum 2 scores

0 0 0 0 0 0 4 4
* * * *

That is, stratum 1 score 0 is 1st, 0 is 3rd, skip digit 0, 0 is 4th, 4 is 8th. So our four without replacement random samples from stratum 1 are 0, 0, 0, 4.

TABLE I Random Numbers

Row \ Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	10480	15011	01536	02011	81647	91646	69179	14194	62590	36207	20969	99570	91291	90700
2	22368	46573	25595	85393	30995	89198	27982	53402	93965	34095	52666	19174	39615	99505
3	24130	48360	22527	97265	76393	64809	15179	24830	49340	32081	30680	19655	63348	58629
4	42167	93093	06243	61680	07856	16376	39440	53537	71341	57004	00849	74917	97758	16379
5	37570	39975	81837	16656	06121	91782	60468	81305	49684	60672	14110	06927	01263	54613
6	77921	06907	11008	42751	27756	53498	18602	70659	90655	15053	21916	81825	44394	42880
7	99562	72905	56420	69994	98872	31016	71194	18738	44013	48840	63213	21069	10634	12952
8	96301	91977	05463	07972	18876	20922	94595	56869	69014	60045	18425	84903	42508	32307
9	89579	14342	63661	10281	17453	18103	57740	84378	25331	12566	58678	44947	05585	56941
10	85475	36857	53342	53988	53060	59533	38867	62300	08158	17983	16439	11458	18593	64952
11	28918	69578	88231	33276	70997	79936	56865	05859	90106	31595	01547	85590	91610	78188
12	63553	40961	48235	03427	49626	69445	18663	72695	52180	20847	12234	90511	33703	90322
13	09429	93969	52636	92737	88974	33488	36320	17617	30015	08272	84115	27156	30613	74952
14	10365	61129	87529	85689	48237	52267	67689	93394	01511	26358	85104	20285	29975	89868
15	07119	97336	71048	08178	77233	13916	47564	81056	97735	85977	29372	74461	28551	90707
16	51085	12765	51821	51259	77452	16308	60756	92144	49442	53900	70960	63990	75601	40719
17	02368	21382	52404	60268	89368	19885	55322	44819	01188	65255	64835	44919	05944	55157
18	01011	54092	33362	94904	31273	04146	18594	29852	71585	85030	51132	01915	92747	64951
19	52162	53916	46369	58586	23216	14513	83149	98736	23495	64350	94738	17752	35156	35749
20	07056	97628	33787	09998	42698	06691	76988	13602	51851	46104	88916	19509	25625	58104
21	48663	91245	85828	14346	09172	30168	90229	04734	59193	22178	30421	61666	99904	32812
22	54164	58492	22421	74103	47070	25306	76468	26384	58151	06646	21524	15227	96909	44592
23	32639	32363	05597	24200	13363	38005	94342	28728	35806	06912	17012	64161	18296	22851
24	29334	27001	87637	87308	58731	00256	45834	15398	46557	41135	10367	07684	36188	18510
25	02488	33062	28834	07351	19731	92420	60952	61280	50001	67658	32586	86679	50720	94953
26	81525	72295	04839	96423	24878	82651	66566	14778	76797	14780	13300	87074	79666	95725
27	29676	20591	68086	26432	46901	20849	89768	81536	86645	12659	92259	57102	80428	25280
28	00742	57392	39064	66432	84673	40027	32832	61362	98947	96067	64760	64584	96096	98253
29	05366	04213	25669	26422	44407	44048	37937	63904	45766	66134	75470	66520	34693	90449
30	91921	26418	64117	94305	26766	25940	39972	22209	71500	64568	91402	42416	07844	69618
31	00582	04711	87917	77341	42206	35126	74087	99547	81817	42607	43808	76655	62028	76630
32	00725	69884	62797	56170	86324	88072	76222	36086	84637	93161	76038	65855	77919	88006
33	69011	65795	95876	55293	18988	27354	26575	08625	40801	59920	29841	80150	12777	48501
34	25976	57948	29888	88604	67917	48708	18912	82271	65424	69774	33611	54262	85963	03547
35	09763	83473	73577	12908	30883	18317	28290	35797	05998	41688	34952	37888	38917	88050

(continued)

TABLE I Continued