1. A population of resistors has 20% that are below standard.

a. Sketch the approximate distribution of \( \hat{p} \), the fraction of below standard resistors in a random with-replacement sample of 100 resistors.

\[
\hat{p} = 0.2 \\
\text{SD} \hat{p} = \sqrt{\frac{p(1-p)}{n}} = \frac{0.4}{10} = 0.04
\]

b. Overlay on (a) the corresponding sketch if the population of resistors numbers 500 and the sampling is WITHOUT replacement.

\[
\text{NARROW SD IN (a) TO } \sqrt{\frac{500}{500 - 1}} (0.04)
\]

c. Determine the z-approximation of \( P(\hat{p} < 0.22) \) (first find \( z \)).

\[
z = \frac{0.22 - \hat{p}}{0.04} = \frac{0.22 - 0.2}{0.04} = 0.5 \\
z \approx \sqrt{0.5} \Rightarrow \left( \frac{2}{0.5} \right) = 0.6915
\]

2. A random with-replacement sample of 40 electronic devices from production finds 13 that are of high grade. Estimate the following:

a. population fraction \( \hat{p} \) of high-grade devices

\[
\hat{p} = \frac{13}{40}
\]

b. population mean \( \mu \) of scores

\[
\mu = E[X] = p(1-p)0 = p \approx 0.325
\]

\[
\text{Estimate } \mu \approx \frac{13}{40}
\]

c. sd of \( \hat{p} \)

\[
\text{SD} \hat{p} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.325(1-0.325)}{40}} = \frac{0.22}{40}
\]

d. Determine a 90% (not 95%) z-based CI for population \( p \).

\[
\text{DOES NOT APPLY - WE'LL USE } z = 1.645 \\
\text{BUT TABLE IV SAYS } 1.684 \text{ (DOH) BE OFF FOR } n = 40
\]

\[
90\% \text{ (APPROX) BASED CI } \hat{p} \pm 1.645 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} = \frac{13}{40} \pm 1.645 \frac{0.22}{40} \\
0.325 \pm 1.645 \frac{0.325}{40}
\]

e. What claim is made for the 90% CI?

\[
P(\hat{p} \in 0.325 \pm 1.645 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}) \in 0.95
\]

f. Determine the estimated margin of error for \( \hat{p} \).

\[
\text{ESTIMATED ERROR OF } \hat{p} \text{ IS } 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} = 1.96 \frac{\sqrt{0.325}}{\sqrt{40}}
\]
3. \( P(\text{OIL}) = 0.2, P(+ \mid \text{OIL}) = 0.7, P(+ \mid \text{OIL}) = 0.3. \)
   a. Make a complete tree diagram including all endpoints such as \( P(\text{OIL}+) \).
   Label everything.
   b. Make a complete Venn diagram.
   c. Determine \( P(\text{OIL} \mid +) \).
   \[
   \frac{P(\text{OIL}+)}{P(+)} = \frac{(0.2 \times 0.7)}{(0.2 \times 0.7 + 0.8 \times 0.3)}
   \]
4. One of 10000 persons has a rare disease. A test is available for which the false positive rate is only 3%. A randomly selected person has tested positive. \( P(+) \mid D^c) = 0.3 \)
   a. Make a tree diagram with disease vs not disease at the root and testing plus or minus on the downstream branches. Fill in everything you know.
   b. Determine \( P(\text{diseased} \mid +) \) in each of the two cases
   \[
P(+ \mid \text{diseased}) = 0 \quad \Rightarrow \quad P(D+) = \frac{P(D+) \cdot P(+ \mid D)}{P(D+)} = \frac{P(D+) \cdot 0}{P(D+) + P(D^c) \cdot 0} = \frac{0}{0} = 0
   \]
   \[
P(+ \mid \text{diseased}) = 1 \quad \Rightarrow \quad P(D+) = \frac{P(D+) \cdot P(+ \mid D)}{P(D+) + P(D^c) \cdot 0.03} = \frac{0.001 \cdot (1)}{0.001 \cdot (1) + 0.9999 \cdot 0.03} \approx 0
   \]
The answers to (a) and (b) are close.
   c. Is the test useful?
   \[
   \text{No} \quad P(D+) \sim 0 \text{ either way.} \quad \text{(equivalent: } P(D^c+) \sim 1 \text{ with given information)}
   \]
5. A process averages around 4 breakdowns per period. Our process monitor says to stop for inspection and cleaning if we experience more than 7 breakdowns in a given period. Assume that breakdowns follow a Poisson distribution \( p(x) = e^{-\lambda} \frac{\lambda^x}{x!} \). 

a. Determine \( P(X > 0) \) but use 3 for "e" (obviously a bit inexact). Your answer is a number.

\[
1 - P(0) = 1 - e^{-4} \frac{4^0}{0!} = 1 - e^{-4} \approx 1 - \frac{1}{81} = \frac{80}{81}.
\]

b. Because \( \mu = 4 > 3 \) we have agreed to use a normal approximation for the distribution. Recall that for a Poisson distribution the variance is the mean. Sketch the distribution and shade the area corresponding to \( x > 7.5 \) (the region used following the continuity correction when approximating \( P(X > 7.5) \)).

\[\text{Ans: } 0.9579 \approx 0.9401\]

6. 20% of parts are defective. Parts are sampled with-replacement and equal probability. The number of defective parts in a sample \( n \) follows the discrete distribution with

\[ p(x) = \text{binomial coefficient for } (n, x) \cdot 2^x \cdot (1-2)^{n-x}, x = 0, \ldots, n. \]

a. Evaluate \( p(2) \) for \( n = 6 \).

\[
\binom{6}{2} \cdot 2^2 \cdot 0.8^4 = \frac{6 \cdot 2}{2!} \cdot 2 \cdot 0.8^4 = 15 \cdot 2 \cdot 0.8^4
\]

b. Evaluate \( P(X > 2) \) exactly (not a z-approximation which would be inapplicable here because \( n = 6 \) is too small).

\[
1 - P(0) - P(1) - P(2) = 1 - (6) \cdot 2^0 \cdot 0.8^6 - (6) \cdot 2^1 \cdot 0.8^5 - (6) \cdot 2^2 \cdot 0.8^4
\]

\[
= 1 - 0.8^6 - 6 \cdot 2 \cdot 0.8^5 - 15 \cdot 2^2 \cdot 0.8^4
\]

c. How many defective parts are expected (on average) in a sample of 6?

\[
np = 6 \cdot 0.2 = 1.2 \text{ (average)}
\]
7. Box I contains \{R, R, R, G, G, Y\} and Box II contains \{R, G, G, G, Y\}. One box will be chosen, box I with probability 1/3 and box II with probability 2/3. Two balls will be selected WITHOUT replacement from the chosen box.

a. Give \( P(R_1 \ Y_2 \mid \text{box I is chosen}) \).

\[
P(R_1 \ Y_2 | R_1) = \frac{1}{6} \cdot \frac{1}{5}
\]

b. Give \( P(R_1 \ Y_2) \).

\[
P(I) P(R_1 \ Y_2 | I) + P(II) P(R_1 \ Y_2 | II)
\]

\[
= \frac{1}{3} \left( \frac{3}{6} \cdot \frac{1}{5} \right) + \frac{2}{3} \left( \frac{1}{5} \cdot \frac{4}{5} \right)
\]

c. Give \( P(\text{box I} \mid R_1) \).

\[
P(I \mid R_1) = \frac{\frac{1}{3} \left( \frac{3}{6} \cdot \frac{1}{5} \right)}{\frac{1}{3} \left( \frac{3}{6} \cdot \frac{1}{5} \right) + \frac{2}{3} \left( \frac{1}{5} \cdot \frac{4}{5} \right)}
\]

d. Give \( P(\text{box I} \mid R_1 \ Y_2) \).

\[
P(I \mid R_1 \ Y_2) = \frac{\frac{1}{3} \left( \frac{3}{6} \cdot \frac{1}{5} \right)}{\frac{1}{3} \left( \frac{3}{6} \cdot \frac{1}{5} \right) + \frac{2}{3} \left( \frac{1}{5} \cdot \frac{4}{5} \right)}
\]

\[\text{DONE IN (b)}\]

8. 5% of parts are out of spec. Three parts are sampled with replacement.

a. Determine \( P(\text{out1 in2 in3}) \).

\[.05 \cdot .95 \cdot .95\]

\[P(\text{in1 out2 in3}) = .95 \cdot .05 \cdot .95\]

\[P(\text{in1 in2 out3}) = .95 \cdot .95 \cdot .05\]

b. From (a) determine \( P(\text{total of one \"out\" and two \"ins\" in three samples})\).

\[\text{SUM OF ABOVE} = 3 \cdot (0.05) \cdot (0.95)^2 = (3) \cdot 0.05 \cdot 0.95^2 (\text{BINOMIAL})\]

\[\text{correction}\]

c. Sketch the approximate distribution of the number \( X \) of \text{OUTS} from a with-replacement sample of 100. Label the mean and sd of the bell curve.

\[\sqrt{100 \cdot 0.05 \cdot 0.95} \approx \sqrt{5}\]

\[\text{ correction}\]

\[0.05 \cdot 100 = 5 \neq \text{OUT}\]

d. Give \( E \ X \), the expected number of \text{OUTS} from the sample of 100 in (c).

Sketch the Poisson distribution with mean \( E \ X \). Compare with (c).
Calculate:

a. Sample sd s for data \{2, 5, 5\}. Do not reduce.
\[ s = \frac{\sqrt{(2-4)^2 + (5-4)^2 + (5-4)^2}}{3-1} \]

b. \( P(Z \in [-1.22, 2.40]) \) from Z-table.

\[ \begin{align*}
-1.22 & \quad \text{TAKEN DIFFERENCE} \\
2.40 & \\
\end{align*} \]

c. \( E(X^3) \) for the distribution below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( p(x) )</th>
<th>( x^3 p(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.9</td>
<td>.9</td>
</tr>
<tr>
<td>2</td>
<td>.1</td>
<td>( \frac{8 \cdot (.1)}{8} = E(X^3) )</td>
</tr>
</tbody>
</table>

\( E(X^3) = 10 \)

d. Using your answer to (c) evaluate \( E(2(X^3 + 5)) \).
\( 2E(X^3) + 10 \)

10. When a machine begins to squeak it needs lubricating. The times \( T \) between such occurrences are modeled as independent and exponentially distributed with a mean of 1.5 hours \( P(T > t) = e^{-(t/1.5)} \) for every \( t > 0 \).

a. Determine the probability we wait longer than 3 hours for this to happen.
Use \( e \approx 2.718281828 \).
\[ P(T > 3) = e^{-3/1.5} = e^{-2} = (2.718281828)^{-2} \]

b. The process has been running for 5.6 hours without a squeak. Determine the conditional probability that it will run at least 3 more hours without a squeak, conditional on the fact that it has already run 5.6 hours without squeaking. You may invoke the "memory less" property of exponential.

\[ P(T > 5.6 + 3 \mid T > 5.6) = P(T > 3) = e^{-2} \]

c. Determine the probability that a squeak occurs before 2 hours of operation.
\[ 1 - P(T > 2) = 1 - e^{-2/1.5} \]