`mortarair = {5.7, 6.8, 9.6, 10.0, 10.7, 12.6, 14.4, 15.0, 15.3, 16.2, 17.8, 18.7, 19.7, 20.6, 25.0};
dryden = {119, 121.3, 118.2, 124.0, 112.3, 114.1, 112.2, 113.1, 111.3, 107.2, 108.9, 107.8, 111.0, 106.2, 105.0};
Length[mortarair] gives the number of values within mortarair

ListPlot[Table[{mortarair[[i]], dryden[[i]]}, {i, 1, 15}],
AxesLabel -> {"x=mortarair", "y=dryden"}, PlotStyle ->PointSize[0.03]]

This plots the x and y data

```
{x,mortarair = Table[{i, mortarair[[i]]}, {i, 1, 15}];
MatrixForm[xmortair] - This shows mortarair(x values) with the 1st column all ones
in matrix form. This is known as the design matrix. It is presented in this format to take the dot product with dryden to find β₀ and β₁.

This probability model stated in matrix form is y = X β + ε
```

```
betahatmortar = PseudoInverse[xmortair].dryden

{126.249, -0.927622}
```

PseudoInverse is the least squares solver applicable to systems of linear equations. It produces the unique solution of simultaneous linear equations in several variables. The least squares solver uses PseudoInverse to find a least squares solution β₀ and β₁ by taking the dot product of xmortair and dryden.

β₀ and β₁ represent the slope and intercepts.
\[ \hat{\beta}_0 = 126.249; \]
\[ \hat{\beta}_1 = -0.917622 \]
\[ \hat{\gamma} i = \hat{\gamma} = \frac{x_i - \bar{x}}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \]
\[ \frac{y_i - \bar{y}}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \]
\[ \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \]
\[ \hat{\beta} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \]
\[ \hat{\gamma} = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \]
\[ \text{From } \hat{\beta} \text{ and } \hat{\gamma}, \text{ the slope and y-intercept are used to find } \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \]
\[ \text{Show[ListPlot[Table[mortarair[[i]], dryden[[i]], {i, 1, 15}],
AxesLabel -> {"x=mortarair", "y=dryden"}, PlotStyle -> PointSize[0.03]],
Graphics[Line[Table[mortarair[[i]], dryden[[i]], {i, 1, 15}]]]]} \]
\[ \text{The line joins the points } (x_i, \hat{y}_i), \text{ the LS produces fitted values on the line.} \]
\[ \hat{\beta}_0 \text{ is indeed the } y \text{-intercept at } 126.249 \]
\[ \hat{\beta}_1 \text{ is the slope at } -0.917622 \]
\[ \text{The height of the regression line are the previously calculated fitted values. Also, the residuals are the signed vertical gaps between the points of the plot and regression line.} \]
\[ \text{dryenresid} = \text{dryen} - \text{dryenhat} \]
\[ \text{Out[71]} = \{-2.01844, 1.29094, 0.760281, 6.92733, -4.13013, -0.58685, -0.835134, 2.61544, -0.909274, -4.13841, -1.01522, -1.28936, 2.83286, -1.14588, 1.69166 \}
\[ \text{Inverse[Transpose[mortarair].mortarair]} \]
\[ \text{Out[72]} = \{0.50824, -0.309887, \{-0.309887, 0.0213128\} \}
\[ \text{sigmahatsquared} = \frac{15 - 1}{15 - 2} (\text{std[dryenresid]}^2) \]
\[ \text{Out[90]} = 8.64948 \]
\[ \text{MatrixForm[} \text{\[beta\] \_covariance} = \begin{pmatrix} 5.0824 & -0.309887 \\ -0.309887 & 0.0213128 \end{pmatrix} \]
\[ \text{Out[79]} = \text{MatrixForm[} \text{\[var\] \[beta\]} = \text{\text{\text{\[var\] \[beta\]}}} \]
\[ \text{Out[80]} = \text{\text{\text{\[var\] \[beta\]}}} \]
\[ \text{variances and covariances that are estimated by entries of the matrix.} \]
\[ \text{standarderror} = \text{Sqrt[variancebetahat1]} \]
\[ \text{Out[125]} = 0.145989 \]
This use of \( t \) results in an exact CI if provided the measurements \((y_i)\) are from a process with statistical control. 

\[
\text{CI} = \left[ \hat{\beta} - t(\hat{\sigma}/n) \right], \quad t(\hat{\sigma}/n) \text{ for } 95\% \text{ CI, } \alpha = .025
\]

This CI agrees with the one reported on page 477.

There is a minor disagreement between my mean and the book mean. Since the mathematics residuals' mean of squares are less than SAS residuals, SAS must be in error (best squares not achieved by SAS).

This is the normal probability plot for the residuals to give a partial check on the normal errors assumption of the probability model.

The correlation between the independent variable mortarair and the dependent variable dryden, squaring it gives the coefficient of determination or "the fraction of variability accounted for by regression on \( x \)."

\[
\text{Accounted for by regression in } x = \frac{\hat{\beta}^2}{\hat{\beta}^2 + \hat{\beta}^2 + \hat{\beta}^2}
\]

I want to be close to 1. It is interpreted as the fraction of \( y^2 - \hat{\beta}^2 \) accounted for by regression in \( x \).
In[49]:= mortarair = {99, 101.1, 102.7, 103.0, 105.4, 107.0, 108.7, 110.8, 112.1, 112.4, 113.6, 113.8, 115.1, 115.4, 120.0};

In[52]:= dryden = {28.8, 27.9, 27, 25.2, 22.8, 21.5, 20.9, 19.6, 17.1, 18.9, 16.0, 16.7, 13, 13.6, 10.8};

Length[mortarair]

Out[53]= Length of terms

ListPlot[Table[{mortarair[[i]], dryden[[i]]}, {i, 1, 15}], AxesLabel -> {"x=mortarair", "y=dryden"}, PlotStyle -> PointSize[0.03]]

Out[50]=

In[57]:= mxmortarair = Table[{i, mortarair[[i]]}, {i, 1, 15}];

In[58]:= MatrixForm[mxmortarair]

Out[58]=

\[
\begin{pmatrix}
1 & 99 \\
1 & 101.1 \\
1 & 102.7 \\
1 & 103. \\
1 & 105.4 \\
1 & 107. \\
1 & 108.7 \\
1 & 110.8 \\
1 & 112.1 \\
1 & 112.4 \\
1 & 113.5 \\
1 & 113.8 \\
1 & 115.1 \\
1 & 115.4 \\
1 & 120. \\
\end{pmatrix}
\]

\text{design matrix with 1st column 1's}

\text{in order to face the dot product with dryden}

\text{do find } b_0 \text{ and } b_1.

\text{design matrix}

In[59]:= betahatmortar = PseudoInverse[mxmortarair].dryden

Out[59]= \begin{pmatrix}
(118.91, -0.904731)
\end{pmatrix}

In[60]:= Beta0hat = 118.91

Out[60]= 118.91

In[61]:= Beta1hat = -0.904731

Out[61]= -0.904731
\[ \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 \cdot x_i \]


\[ \text{In[83]} = \text{Show[ListPlot[Table[\{mortarair[[i]], dryden[[i]]\}, \{i, 1, 15\}], AxesLabel \rightarrow \{"x=mortarair", "y=dryden"\}, PlotStyle \rightarrow \text{PointSize[0.03]}\}, \text{Graphics[Line[Table[\{mortarair[[i]], dryden[[i]]\}, \{i, 1, 15\}]\]}]]} \]

\[ \text{Out[83]} = \]

\[ \text{drydenresid} = \text{dryden} - \text{drydenhat} \rightarrow \text{pulls out the residuals} \] \[ y_i - \hat{y}_i = y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i) \]

\[ \text{Out[84]} = \{-0.541582, 0.458353, 1.00592, -0.522659, -0.751305, -0.603736, 0.334306, 0.93424, -0.38961, 1.68181, -0.132514, 0.748432, -1.77542, -0.903999, 0.457762 \} \]

\[ \text{In[85]} = \text{Inverse[Transpose[mortarair] \cdot mortarair]} \frac{15 - 1}{15 - 2} (\text{s[drydenresid]}^2) \]

\[ \text{Out[85]} = \{(20.2421, -0.184593), (-0.184593, 0.00168825)\} \]

\[ \text{In[86]} = \text{MatrixForm} \left( \begin{array}{cc} 20.2421 & -0.184593 \\ -0.184593 & 0.00168825 \end{array} \right) \]

\[ \text{Out[86]} = 0.87991 \]

\[ \text{variancebetahat1} = 0.00168825 \]

\[ \text{Out[70]} = 0.00168825 \]

\[ \text{standarderror} = \text{Sqrt[variancebetahat1]} \]

\[ \text{Out[71]} = 0.0410883 \]

\[ \text{degrees freedom} (n-t) = 15-2 = 13 \]

\[ \text{tvalue} = t[13, .95] \]

\[ \text{Out[72]} = 2.16037 \]

\[ \text{ci} = \{\text{Slisthat} - \text{tvalue} \cdot \text{standarderror} \cdot \text{standarderror}, \text{Slisthat} + \text{tvalue} \cdot \text{standarderror} \cdot \text{standarderror}\} \]

\[ \text{Out[73]} = \{-0.993497, -0.815965\} \]
In[76] := mean[(dryden - xmortarair.{118.91, -0.9047306579482643})^2]
Out[76] := 0.762589

In[77] := normalprobabilityplot[drydenresid, 0.03]

Out[77] :=

In[78] := r[mortarair, dryden] := the regular correlation between the y's observed and the fitted values y, \bar{y}.
Out[78] := -0.986857

In[79] := \%^2
Out[79] := 0.973887 per \%^2

y closer to one, is a better correlation.
It is interpreted as a fraction of \( \bar{y} - \bar{y} \) accounted for by the regression.