1. The pmf of r.v. \( X \) is \( p(0) = \frac{1}{3}; p(2) = \frac{2}{3} \). Determine \( E \frac{X+2}{X+1} \).
\[
E \frac{X+2}{X+1} = \sum_{X} \frac{x+2}{x+1} p(x) = \frac{2}{3} \left( \frac{1}{3} \right) + \frac{4}{3} \left( \frac{2}{3} \right)
\]

2. Random variables \( X, Y \) have
\[
E X = 1 \quad \text{sd} X = 2 \quad E Y = 3 \quad \text{sd} Y = 4.
\]
a. Determine \( E(X + 9Y - 8) \)
\[
E X + 9E Y - 8 = 1 + 9(3) - 8 = 20
\]
b. Determine \( \text{sd} (6Y - 15) \)
\[
\sqrt{6^2 \text{sd} Y} = 6 \left( \frac{4}{4} \right)
\]
c. Supposing that \( X, Y \) are independent, determine \( \text{Var}(10X + 30Y + 40) \).
\[
\text{INDEPENDENT } \quad \text{Var} 10X + \text{Var} 30Y = 10^2 2^2 + 30^2 \frac{4^2}{4}
\]

3. \( F \) stands for "casting is faulty"
+ stands for "casting appears to be faulty" etc.
\[
P(F) = 0.1 \quad \text{P(}+|F\text{)} = 0.6 \quad \text{P(}+|\overline{F}\text{)} = 0.2
\]
Determine
a. \( P(+|E) = P(F) P(+|F) + P(\overline{F}) P(+|\overline{F}) = 0.1 \cdot 0.6 + 0.9 \cdot 0.2 \)
b. \( P(F|+) = \frac{P(F|+)}{P(+)} = \frac{0.1 \cdot 0.6}{0.1 \cdot 0.6 + 0.9 \cdot 0.2} \)

4. Tom will draw first with equal probability. Sue will draw second with equal probability on those remaining from \{5 5 5 5 1 1 3 3\}
a. \( P(\text{Sue 5} \mid \text{Tom}) \) (per description, intuitive)
\[
\frac{\text{Draw from 5 5 5 5 1 1 3 3}}{\text{Tom}} = \frac{2}{8}
\]
b. \( P(\text{Sue 5}) \) using total probability and mult rules (show your work, do not reduce).
\[
P(5 5 5 5) = P(5 5 5 5) = P(5 5 5 5) + P(5 5 5 5) + P(5 5 5 5)
\]
c. \( P(\text{Tom or Sue get 5}) \) (show your work, do not reduce)
\[
P(\text{Tom or Sue get 5}) = P(5 5 5 5) + P(5 5 5 5) - P(5 5 5 5)
\]

5. Business receipts average 12,677 per day with \( \text{sd} 6,903 \). Receipts on different days seem to be independent. Sketch the approximate distribution of \( T = \text{total receipts from 400 such days} \). Be sure to evaluate and display in your sketch \( E T \) and \( \text{sd} T \). Show appropriate reasoning.
\[
E T = E(X_1 + \cdots + X_{400}) = 400 E X = 400 \left( \frac{12,677}{400} \right)
\]
\[
\text{Var} T = \text{Var}(X_1 + \cdots + X_{400}) = \text{Var} X = 400 \frac{6,903^2}{400}
\]

6. The Poisson distributed number of bad microchips averages around 9 per batch.
b. How unusual is it to find 6 or fewer bad chips in a batch? (see (a) below)
\[
\frac{6 - 9}{3} = -1.5 \quad \approx 1.3 \quad \text{below mean} \quad -1.5 \text{SD}
\]
a. Sketch the approximate dist of number of bad chips in a batch, labeling mean, sd.
1. The pmf of r.v. X is \( p(0) = 1/3, p(-2) = 1/3, p(2) = 1/3 \). Determine \( E X^3 \).

\[
E X^3 = \sum x^3 p(x) = 0^3(\frac{1}{3}) + (-2)^3(\frac{1}{3}) + 2^3(\frac{1}{3}) = 0 \quad \text{(You need not reduce to 0)}
\]

2. Random variables X, Y have

\[E X = 2 \quad \text{sd} X = 3 \quad E Y = 4 \quad \text{sd} Y = 5.\]

a. Determine \( E(6X - Y + 1) = 6E X - E Y + 1 = 6(2) - 4 + 1 = 9 \)

b. Determine \( \text{sd} (11X + 3) = |11| \text{sd} X = 11(3) = 33 \)

c. Supposing that X, Y are independent, determine \( \text{Var}(20X + 15Y + 13) \).

\[
\text{Var}(20X + 15Y) = 20^2 \text{Var} X + 15^2 \text{Var} Y = 20^2 \cdot 3^2 + 15^2 \cdot 5
\]

3. D stands for "person has the disease" + stands for "person tests positive for the disease" etc.

\[P(D) = 0.2 \quad P(+) = 0.6 \quad P(+) \mid D^c = 0.1\]

Determine

a. \( P(+) = P(D+) + P(D^c+) = P(D)P(+ \mid D) = 0.2 \cdot 0.6 + 0.1 \)

b. \( P(D \mid +) = \frac{P(D+) \mid P(+) = 0.2 \cdot 0.6}{0.2 \cdot 0.6 + 0.8 \cdot 0.1} \)

4. \( P(A) = 0.5, P(B) = 0.4, P(B \mid A) = 0.2 \). Determine but do not reduce:

a. \( P(AB) = P(A) \cdot P(B \mid A) = 0.5 \cdot 0.2 = 0.1 \)

b. \( P(A \cup B) = P(A) + P(B) - P(AB) = 0.5 + 0.4 - 0.1 = 0.8 \)

c. Are A, B independent? Show your reasoning!

\[P(B \mid A) = P(A) ? \quad \text{No, } 0.2 \neq 0.5 \Rightarrow A, B \text{ ARE NOT INDEPENDENT.} \]

5. A random sample of 100 vehicles is selected with replacement and with equal probability from a fleet whose mpg average 15.5 with sd 4.3. Sketch the approximate distribution for \( T = \text{total mpg of all 100 sample vehicles}. \) Be sure to evaluate and display in your sketch \( E T \) and \( \text{sd} T \). Show appropriate reasoning.

\[E T = E (X_1 + \cdots + X_{100}) = 100 E X = 1550\]

\[\text{Var} T = \text{Var}(X_1 + \cdots + X_{100}) = 100 \text{Var} X = 100 \cdot 4.3^2 \]

6. The Poisson distributed number of bad microchips averages around 9 per batch.

b. How unusual is it to find 12 or more bad chips in a batch? (see (a) below)

\[12 - 9 = +3 \quad \text{SD} \quad 16^9 \text{ ABOVE mean} + 1.0 \text{ SD} .\]

a. Sketch the approximate dist of number of bad chips in a batch, labeling mean, sd.