EXAM 1 KEY

1. The pmf of r.v. X is given below. Determine the requested quantities.

x	p(x)	x p(x)	x^2 p(x)
0	1/4	0	0
2	2/4	1	2
4	1/4	1	4

a. E X

$= 0 + 1 + 1 = \boxed{2}$

b. E (X^2)

$= 0 + 2 + 4 = \boxed{6}$

c. Var X

$= E(X^2) - (EX)^2$
$= 6 - (2)^2$
$= 6 - 4 = \boxed{2}$

d. sd X

$\boxed{\sqrt{2}}$

e. E(4 X - 5) $= 4EX - 5$
$= 4(2) - 5$
$= 8 - 5 = \boxed{3}$

f. Var(4 X - 5)

$= 4^2 \, \mathrm{Var} X$
$= 16(2) = \boxed{32}$

g. sd(4 X - 5)

$= \sqrt{\mathrm{Var}(4X-5)} = 4 \, \mathrm{sd}X = \boxed{4\sqrt{2}}$

h. E (1/X)

$= \left(\frac{1}{0}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) =$ undefined

2. For the list {4, 5, 6, 8} determine

a. Height of the probability histogram for the class interval [3.5, 5.5).

$\frac{\left(\frac{2}{4}\right)}{2} = \frac{1/2}{2} = 1/4$

b. Sample standard deviation s. $\bar{x} = \frac{4+5+6+8}{4} = \frac{23}{4}$

$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{(4-23/4)^2+(5-23/4)^2+(6-23/4)^2+(8-23/4)^2}{3}}$

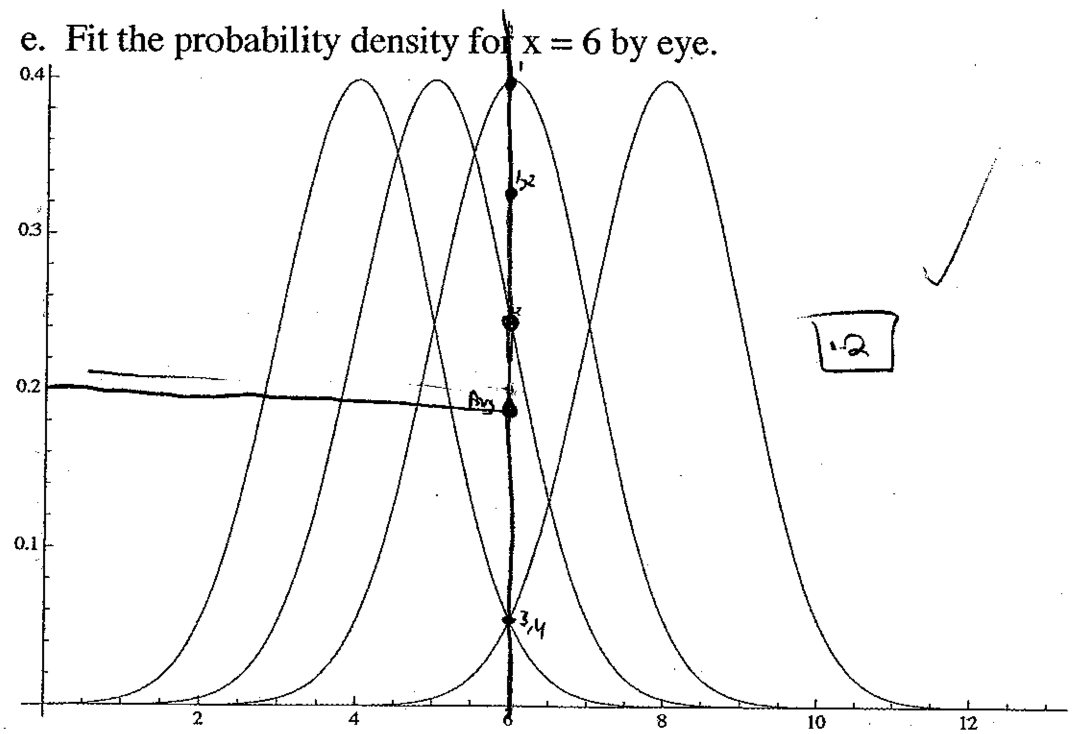
c. Margin of error for the sample mean $\bar{x}$.

$1.96\left(\frac{\sqrt{\frac{(4-23/4)^2+(5-23/4)^2+(6-23/4)^2+(8-23/4)^2}{3}}}{\sqrt{4}}\right)$

d. Around what percentage of with-replacment equal-probability random samples X_1, ..., X_n yield a confidence interval $\overline{X} \pm 1.0 \frac{s}{\sqrt{n}}$ which covers the population mean μ?

68%

e. Fit the probability density for x = 6 by eye.



1.2

3. **P(A) = 0.5, P(B | A) = 0.2, P(B | A^C) = 0.4.**

a. P(AB) $= P(A)\cdot P(B|A)$
$\boxed{= (.5)(.2)}$

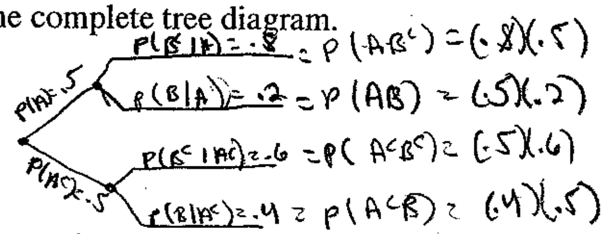
b. P(A^CB) $= P(A^c)\cdot P(B|A^c)$
$\boxed{= (.5)(.4)}$

c. P(B)
$= P(AB) + P(A^cB)$
$= (.5)(.2) + (.5)(.4) = .1 + .2 = .3$

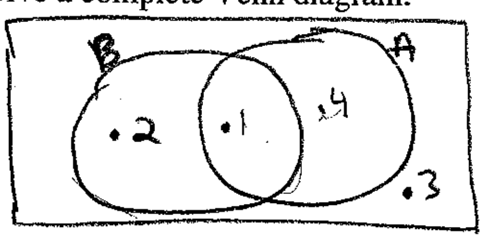
d. Are A, B independent? Show your reasoning!

no, because $P(B) \neq P(B|A)$ and $P(B) \neq P(B|A^c)$

e. Give the complete tree diagram.



f. Give a complete Venn diagram.



g. P(A | B) $= \frac{P(AB)}{P(B)} = \frac{(.5)(.2)}{(.5)(.2)+(.4)(.5)} = \boxed{\frac{.1}{.3}}$

4. **Box 1 will be chosen with probability 1/3. Otherwise box II will be chosen. We will then draw two balls from the chosen box, without replacement and with equal probability on those remaining.**

Box I {R R R R G Y Y Y} (8 in all), **Box II** {R R G G G Y} (6 in all)

a. P(R1 | I) $= \boxed{4/8}$

b. P(R1) $= \left(\frac{1}{3}\right)\left(\frac{4}{8}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{6}\right)$

40
.2
8.0

5. Business ventures have random (gross) returns X, Y respectively. These are independent with

E X = 10 sd X = 20 E Y = 40 sd Y = 30

Net profit is defined = .3(X - 4) + .2(Y-6).

= .3x − 1.2 + .2y − .12 = .3x + .2y − 2.4

a. E (net profit)

.3 EX − 1.2 + .2EY − 1.2 = $\boxed{.3(10) + .2(40) - 2.4 = 3 + 8 - 2.4}$

b. Var(net profit) = .3² Var x + .2² Var y Var x = 20² Var y = 30²

$\boxed{= (.3^2)(20^2) + .2^2(30^2)}$

6. A telephone sales organization finds that hourly return X averages \$3,458 with standard deviation \$3,332. We have 50 independent such hourly returns.

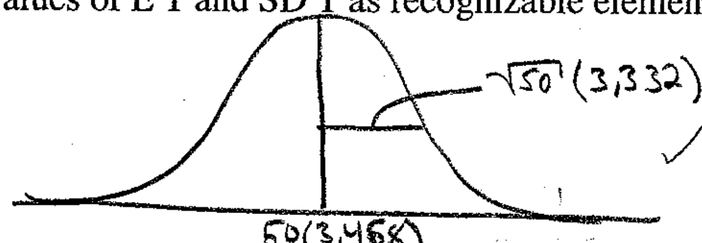
T = $x_1 + x_2 + \cdots x_{50}$ indep. hourly returns

a. E T

= 50(3,458)

b. Var T = 50 (3,332²)

c. Sketch the approximate distribution of their total T = X_1, ..., X_{50}. Identify the numerical values of E T and SD T as recognizable elements in your sketch.

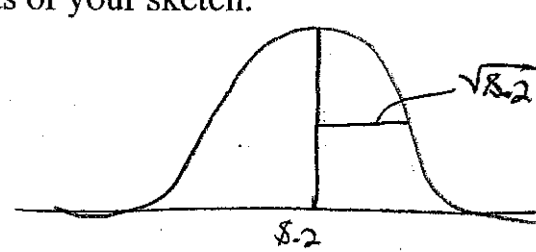


7. We average 8.2 flaws in a given length of fabric. The number of flaws is a Poisson distributed random variable X. The pmf is p(x) = $e^{-8.2}$ 8.2^x / x! for x = 0, 1, 2, ...

a. P(X > 1) = 1 − (p(0) + p(1)) = $\boxed{1 - \left(e^{-8.2}\left(\frac{8.2^0}{1} + \frac{8.2^1}{1}\right)\right)}$

b. sd X EX = 8.2 sd x = $\sqrt{EX}$ = $\boxed{\sqrt{8.2}}$

c. Sketch the approximate distribution of X showing the numerical mean and sd of X as recognizable elements of your sketch.



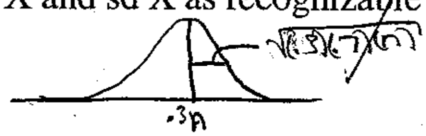
8. Each given item from production has probability 0.3 of being defective. The pmf for X = the number defective items in a random sample of n is p(x) = $\binom{n}{x}$ $0.3^x 0.7^{n-x}$, x = 0, 1, 2, ..., n.

$= \frac{n!}{x!(x-n)!}$ np

a. E X = $\boxed{.3n}$

b. Var X = sd = $\sqrt{np(1-p)}$ Var X = sd^2 = np(1−p) = .3n(.7)

c. Sketch the approximate distribution of X for large n. Be sure to identify the numerical values of E X and sd X as recognizable elements in your sketch.



9. Red die has the numbers {3 4 5 20 21 22}.
Green die has numbers {6 7 8 9 23 24}.

If the two are thrown what is the probability that the red die turns up a larger number than the green die?

Red

	3	4	5	20	21	22
6	0	0	0	1	1	1
7	0	0	0	1	1	1
8	0	0	0	1	1	1
Green 9	0	0	0	1	1	1
23	0	0	0	0	0	0
24	0	0	0	0	0	0

= $\boxed{12/36}$