1. The pmf of r.v. X is given below. Determine the requested quantities.

<table>
<thead>
<tr>
<th>x</th>
<th>p(x)</th>
<th>x p(x)</th>
<th>x^2 p(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2/4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1/4</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{a. } E(X) & = 0 \cdot \frac{1}{4} + 1 \cdot \frac{2}{4} + 4 \cdot \frac{1}{4} = 2 \\
\text{b. } E(X^2) & = 0 \cdot \frac{1}{4} + 1 \cdot \frac{2}{4} + 4 \cdot \frac{1}{4} = 1 \\
\text{c. } \text{Var}(X) & = E(X^2) - (E(X))^2 = 1 - \left(\frac{2}{2}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2} \\
\text{d. } \text{sd}(X) & = \sqrt{\text{Var}(X)} = \sqrt{\frac{1}{2}} = \sqrt{\frac{1}{2}} \\
\text{e. } E(4X - 5) & = 4E(X) - 5 = 4 \cdot \frac{2}{2} - 5 = 2 - 5 = 3 \\
\text{f. } \text{Var}(4X - 5) & = 4^2 \times \text{Var}(X) = 16 \cdot \frac{1}{2} = 8 \\
\text{g. } \text{sd}(4X - 5) & = \sqrt{\text{Var}(4X - 5)} = \sqrt{8} = \sqrt{8} \\
\text{h. } E\left(\frac{1}{X}\right) & = \left(\frac{1}{2}\right) \left(\frac{1}{4}\right) + \left(\frac{2}{4}\right) \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right) \left(\frac{1}{4}\right) = \text{undefined}
\end{align*}
\]
2. For the list \{4, 5, 6, 8\} determine

a. Height of the probability histogram for the class interval [3.5, 5.5].
\[
\sqrt{\frac{3.5}{2}} = \frac{1.75}{1} = 1.75
\]

b. Sample standard deviation \( s \).
\[
s = \sqrt{\frac{(4-5)^2 + (5-5)^2 + (6-5)^2 + (8-5)^2}{n-1}}
\]

\[
s = \sqrt{\frac{9 + 0 + 1 + 9}{3}} = \sqrt{\frac{19}{3}}
\]

c. Margin of error for the sample mean \( \bar{x} \).
\[
1.96 \left( \frac{(4-5)^2 + (5-5)^2 + (6-5)^2 + (8-5)^2)}{\sqrt{3}} \right)
\]

d. Around what percentage of with-replacement equal-probability random samples \( X_1, \ldots, X_n \) yield a confidence interval \( \bar{X} \pm 1.0 \frac{s}{\sqrt{n}} \) which covers the population mean \( \mu \)?

\[
\frac{1.0 \sqrt{3}}{\sqrt{n}}
\]

e. Fit the probability density for \( x = 6 \) by eye.
3. \( P(A) = 0.5, P(B \mid A) = 0.2, P(B \mid A^C) = 0.4. \)

a. \( P(AB) = P(A) \cdot P(B \mid A) \)
\[
= (0.5)(0.2)
\]

b. \( P(A^C B) = P(A^C) \cdot P(B \mid A^C) \)
\[
= (0.5)(0.4)
\]

c. \( P(B) = P(AB) + P(A B^C) \)
\[
= (0.5)(0.2) + (0.5)(0.4) = 0.1 + 0.2 = 0.3
\]

d. Are \( A, B \) independent? Show your reasoning!

No, because \( P(B) \neq P(B \mid A) \) and \( P(B) \neq P(B \mid A^C) \)

e. Give the complete tree diagram.

f. Give a complete Venn diagram.

\[
P(A \mid B) = \frac{P(AB)}{P(B)} = \frac{(0.5)(0.2)}{(0.5)(0.2) + (0.5)(0.4)} = \frac{0.1}{0.3}
\]

4. Box I will be chosen with probability 1/3. Otherwise box II will be chosen. We will then draw two balls from the chosen box, without replacement and with equal probability on those remaining.

Box I \( \{R\ R\ R\ G\ Y\ Y\ Y\} \) (8 in all), Box II \( \{R\ R\ G\ G\ G\ Y\} \) (6 in all)

a. \( P(R1 \mid I) = \frac{1}{8} \)

b. \( P(R1) = \left(\frac{1}{3}\right)\left(\frac{1}{8}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{6}\right) \)
5. Business ventures have random (gross) returns $X$, $Y$ respectively. These are independent with

$$EX = 10 \quad sd \, X = 20 \quad EY = 40 \quad sd \, Y = 30$$

**Net profit** is defined as $3(X - 4) + 0.2(Y - 6)$.

$$z = 3x - 1.2 + 0.2y - 1.2 = 3x + 0.2y - 2.4$$

a. $E(\text{net profit}) = 3\, EX - 1.2 + 0.2\, EY - 1.2 = 3\, (10) + 0.2\, (40) - 2.4 = 3 \cdot 8 - 2.4$

b. $\text{Var(\text{net profit})} = 0.2^2 \, \text{Var} \, X + 0.2^2 \, \text{Var} \, Y$

$$= (0.2^2 \cdot (3 \, \text{SD}^2) + 0.2^2 \cdot (3 \, \text{SD}^2))$$

6. A telephone sales organization finds that hourly return $X$ averages $\$3,458$ with standard deviation $\$3,332$. We have 50 independent such hourly returns.

$T = X_1 + X_2 + \ldots + X_{50}$, $X_i$'s: hourly returns

a. $E(T) = 50(3.458)$

b. $\text{Var}(T) = 50(3.332^2)$

c. Sketch the approximate distribution of their total $T = X_1, \ldots, X_{50}$. Identify the numerical values of $E(T)$ and $\text{SD}(T)$ as recognizable elements in your sketch.

7. We average 8.2 flaws in a given length of fabric. The number of flaws is a Poisson distributed random variable $X$. The pmf is $p(x) = e^{-8.2} \frac{8.2^x}{x!}$ for $x = 0, 1, 2, \ldots$.

a. $P(X > 1)$

$$= 1 - (p(0) + p(1)) = 1 - \left( e^{-8.2} \frac{8.2^0}{0!} + \frac{8.2^1}{1!} \right)$$

b. $sd \, X \leq 8.2 \quad sd \, X = \sqrt{EX} = \sqrt{8.2}$

c. Sketch the approximate distribution of $X$ showing the numerical mean and $sd$ of $X$ as recognizable elements of your sketch.
8. Each given item from production has probability 0.3 of being defective. The pmf for $X$ = the number defective items in a random sample of $n$ is $p(x) = \binom{n}{x} 0.3^x 0.7^{n-x}$, $x = 0, 1, 2, ..., n$.

   a. $E(X) = \frac{np}{n(n-1)}$, $np$ = \[ \begin{array}{c} \frac{n!}{(n-x)!x!} \end{array} \]

   b. $\text{Var}(X) = np(1-p) = .3n(1.7)$

   c. Sketch the approximate distribution of $X$ for large $n$. Be sure to identify the numerical values of $E(X)$ and $\text{sd}(X)$ as recognizable elements in your sketch.

9. Red die has the numbers $\{3 \ 4 \ 5 \ 20 \ 21 \ 22\}$.
   Green die has numbers $\{6 \ 7 \ 8 \ 9 \ 23 \ 24\}$.

If the two are thrown what is the probability that the red die turns up a larger number than the green die?