HW 2. Due at the close of class Monday, September 8, 2008. You will have the oportunity to ask questions in class but try to get as much done before class as you can.

Leave answers as unevaluated fractions but show work and plug in all numerical quantities. We want to see that you know what to do.

1. A box of balls [R R R R G G Y Y Y]. Draws will be selected with equal probability and without replacement from those remaining in the box.

a. P(R1) $\boxed{\frac{4}{9}}$

b. P(R2) by "order of the deal does not matter." $\boxed{\frac{4}{9}}$

c. P(R2) by use of the Law of Total Probability coupled with the Multiplication Rule. Reduce to confirm your answer to (b).

$P(R1 \cap R2) = P(R1)\,P(R2 \mid R1)$

$P(R1) = P(R1R2) + P(R1R2^c)$

$(4/9)(3/8) = \boxed{\frac{1}{6}}$

$(4/9)(3/8) + (4/9)(5/8) = 4/9$

d. P(R4 | R1 R2 Y3)
Write out the box from which the fourth ball is selected under the condition R1 R2 Y3. $\boxed{\frac{1}{3}}$ $\boxed{RRGGYY}$

e. P(R1 or R2) = P(R1 ∪ R2) using the addition rule.

$P(R1) + P(R2) - P(R1R2)$

$(4/9) + (4/9) - (4/9)(3/8) = \boxed{\frac{13}{18}}$

f. P(R1 or R2) if draws are instead with replacement.

$(4/9) + (4/9) - (4/9)(4/9) = \boxed{56/81}$

2. P(Woody's has ~~Samosa~~ Kusa tomorrow) = 0.97.

a. What is

P(Woody's has Samosa tomorrow | my wife awakes after 11 a.m. tomorrow).
Give a reasonable answer and explain your reasoning.

The two events are not related at all, they are independant.

So probability is still .97

b. If in (a) we replace my wife with the lady who freshly prepares the Samosa in the morning would your argument (a) still hold?

P(Kusa tomorrow | lady who prepares) = 0, she didn't do it if she wasn't up

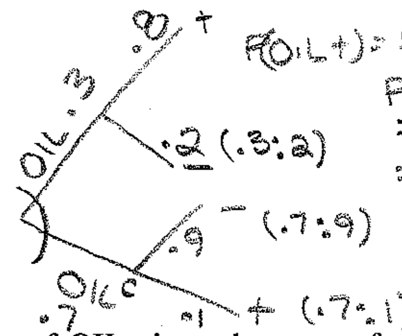
3. We are given
   P(OIL) = 0.3 (30% chance oil is present in the field)
   P(+ | OIL) = 0.8 (80% chance a test for oil is positive given oil is present)
   P(+ | no OIL) = 0.1 (10% chance a test for oil is positive if oil is not present)

Determine

a. P(no OIL) $1 - .3 = \boxed{.7}$

b. P(- | OIL) $1 - .8 = \boxed{.2}$

c. Fill out a complete tree diagram for the given information.



P(OIL+) = P(OIL) × P(+|OIL) = .3·.8 = .24

d. Use the tree diagram to compute P(OIL | +) (the revised chance of OIL given that a test for OIL is positive). This may also be computed according to

P(OIL | +) = P(OIL +) / P(+) (same as multiplication rule rearranged)
= P(OIL +) / (P(OIL +) + P(no OIL +)) (using multiplication rule)

This is Bayes' Formula. One tests for OIL then revises the chance of OIL in the light of a positive test outcome.

$$P(OIL \mid +) = \frac{P(OIL+)}{P(+)} = \frac{.24}{.24 + (.7)(.1)} = \boxed{.77}$$

4. Box [5 5 1 1 1]. Jack draws first with equal probability. Jill draws second from the four bills remaining, with equal probability.

a. P(Ja 5) $\boxed{\frac{2}{5}}$

b. P(Ji 5) by using the rules. Compare with P(Ja 5).

$\boxed{\frac{2}{5}}$

5. The two engines of a plane fail **independently**. Each engine fails with probability 0.01 on a given flight.

a. P(both fail)

$(.01)(.01) = \boxed{.0001}$

b. P(neither fails)

$(.99)(.99) = .9801$

c. P(second fails | first fails)

$\boxed{.01}$

d. P(at least one engine fails)

$P(A) + P(B) - P(A \text{ and } B) = .01 + .01 - (.01)(.01) = \boxed{.0199}$

6. P(rain today) = 0.8, P(rain tomorrow) = 0.6, P(rain both days) = 0.2.

Show that the above information cannot be correct.

P(both) = P(rain tom | rain today) P(rain today), events are not dependant, $\boxed{(.8)(.6) = .48 \neq .2}$

7. One die has numbers 1, 4, 6, 6, 9, 12 on its six faces. Another die has the numbers 2, 5, 7, 8, 8, 14. If the two dice are thrown together what is the probability that the first die throws the larger number? Enumerate all 36 possible outcomes and check off those that favor die one.

(1,2)	(1,14)	(4,8)	(6,8)	✓(9,7)	✓(12,5)
(1,5)	✓(4,2)	(4,14)	(6,8)	✓(9,8)	✓(12,7)
(1,7)	(4,5)	✓(6,2)	(6,14)	✓(9,8)	✓(12,8)
(1,8)	(4,7)	✓(6,5)	✓(9,2)	(9,14)	✓(12,8)
(1,8)	(4,8)	(6,7)	✓(9,5)	✓(12,2)	(12,14)

x2

$\boxed{\frac{15}{36}}$

8. You and I each guess the height of a tree on campus. Our model supposes us to be equally good guessers and (arranging our numerical guesses in order of size in relation to the true height) is

me you true (i.e. I am low, so are you but you are closer to the true height) M Y T
you me true Y M T ✓
true me you T M Y ✓
true you me T Y M
me true you (i.e I am low, you are high, but I am closer to the true height) M T Y ✓
me true you M T Y ✓
you true me Y T M ✓
you true me Y T M ✓

Our model regards these eight as equally probable. But I am a "second guesser," never announcing my actual numerical guess. Instead, I simply wait for you to guess and privately note whether your guess exceeds my own. If it does I only say "you are too high." If it does not I say "you are too low." With what probability am I correct? Check off the favorable cases. Lots of people try to score points by adopting this practice. They seem smart because they are often right.

$\frac{6}{8}$ or $\boxed{\frac{3}{4}}$