

Name : SOLUTIONS

Due Wednesday, August 1, 2019

Instruction: Print this file. Write the answer on a dotted line if provided. Show your work in the space below a question. **Not justified answers earn no points (even if they are correct).** Attach extra pages if needed. Staple everything together. You may use any calculator and any computer software to solve the problems. Number of points for each problem is in square brackets []. Total for this homework is 20 points, so it includes 3 bonus points.

1. A certain market has both an express checkout line and a superexpress checkout line. Let X_1 denote the number of customers in line at the express checkout at a particular time of day, and let X_2 denote the number of customers in line at the super express checkout at the same time. Suppose the joint pmf of X_1 and X_2 is as given in the accompanying table.

		x_2			
		0	1	2	3
x_1	0	.08	.07	.04	.00
	1	.06	.15	.05	.04
	2	.05	.04	.10	.06
	3	.00	.03	.04	.07
	4	.00	.01	.05	.06

- a. [1] What is the probability that there are total of at least four customers in the two lines ?

ANS: 0.46

- b. [1] Determine the marginal pmf of X_1 , and then calculate the expected number $E(X_1)$ of customers in line at the express checkout.

x_1	$P(X_1)$
0	0.19
1	0.30
2	0.25
3	0.14
4	0.12

$E(X_1) = 1.70$

ANS: $E(X_1) = 1.70$

- c. [1] Determine the marginal pmf of X_2 , and then calculate the expected number $E(X_2)$ of customers in line at the super express checkout.

x_2	0	1	2	3
$P(X_2)$	.19	.30	.28	.23

$E(X_2) = 1.55$

ANS: $E(X_2) = 1.55$

d. [1] Are X_1 and X_2 independent random variables? Explain

NO, FOR EXAMPLE $P(X_1=4, X_2=0) = 0 \neq P(X_1=4)P(X_2=0) = 0.12 \cdot 0.19$

e. [1] Compute $\text{Cov}(X_1, X_2)$

ANS: 0.695

$$E(X_1 X_2) = \sum_{x_1=0}^4 \sum_{x_2=0}^3 x_1 x_2 P(X_1, X_2) = 3.33$$

$$\text{Cov}(X_1, X_2) = E(X_1 X_2) - E(X_1)E(X_2) = 3.33 - 1.70 \cdot 1.55 = 0.695$$

2. Each front tire on a particular type of vehicle is supposed to be filled to a pressure of 26 psi. Suppose the actual air pressure in each tire is a random variable: X for the right tire and Y for the left tire, with joint pdf

$$f(x, y) = \begin{cases} K(x^2 + y^2) & 20 \leq x \leq 30, 20 \leq y \leq 30 \\ 0 & \text{otherwise} \end{cases}$$

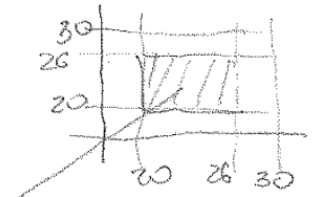
a. [1] What is the value of K ? Show your work!

ANS: $3/380000 = 0.00000789475$

$$K \int_{20}^{30} \int_{20}^{30} (x^2 + y^2) dx dy = 380,000 \frac{K}{3} = 1 \Rightarrow K = \frac{3}{380,000}$$

b. [2] What is the probability that both tires are underfilled? Show your work!

ANS: 0.3024



$$P(X \leq 26, Y \leq 26) = \frac{3}{380,000} \int_{20}^{26} \int_{20}^{26} (x^2 + y^2) dx dy = \frac{3}{380,000} \cdot 38304$$

c. [1] Determine the (marginal) pdf's $f_X(x)$ of air pressure in the right tire alone. Show your work!

$$f_X(x) = \int_{20}^{30} K(x^2 + y^2) dy = \frac{3}{380,000} x^2 + \frac{1}{20}, \quad 20 \leq x \leq 30$$

3. Let X_1 , X_2 , and X_3 represent the times necessary to perform three successive repair tasks at a certain service facility. Suppose they are independent, normal rv's with expected values $\mu_1 = 60$, $\mu_2 = 50$, $\mu_3 = 40$, and standard deviations $\sigma_1 = 15$, $\sigma_2 = 10$, $\sigma_3 = 5$, respectively. Let $Y = X_1 + 2X_2 - 3X_3$

- a. [2] Find the expected value $E(Y)$ and the variance $V(Y)$ of Y .

$$60 + 2 \cdot 50 - 3 \cdot 40 = 40$$

$$15^2 + 4 \cdot 10^2 + 9 \cdot 5^2 = 850$$

$$\text{ANS: } E(Y) = 40, \quad V(Y) = 850$$

- b. [1] Assuming additionally that the random variables X_1 , X_2 , and X_3 are independent and that each is normally distributed, compute the probability $P(X_1 + 2X_2 - 3X_3 > 60)$

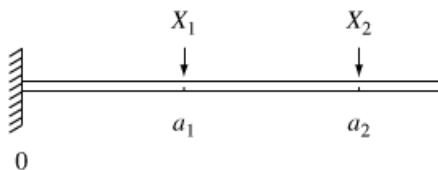
$$\text{Answer: } 0.2464$$

$$Y = X_1 + 2X_2 - 3X_3 \text{ is normal}$$

$$\text{with } \mu_Y = 40, \quad \sigma_Y^2 = 850.$$

$$\text{Hence } P(Y > 60) = \text{normalcdf}(60, 10^{99}, 40, \sqrt{850}) = 0.2464$$

4. [2] If two loads are applied to a cantilever beam as shown in the accompanying drawing, the bending moment at 0 due to the loads is $M = a_1X_1 + a_2X_2$.



- a. Suppose that X_1 and X_2 are independent rv's with means 2 and 4 kips, respectively, and standard deviations .5 and 1.0 kip, respectively. If $a_1 = 5$ ft and $a_2 = 10$ ft, what is the expected bending moment and what is the variance of the bending moment?

$$\text{Answer: } E(M) = 50$$

$$V(M) = 106.25$$

$$M = 5X_1 + 10X_2$$

$$E(M) = 5 \cdot 2 + 10 \cdot 4 = 50$$

$$V(M) = 5^2 \cdot 0.5^2 + 10^2 \cdot 1^2 = 106.25$$

- b. If X_1 and X_2 are normally distributed, what is the probability that the bending moment will exceed 75 kip-ft?

$$\text{Answer: } 0.008$$

$$P(M > 75) = \text{normalcdf}(75, 10^{99}, 50, \sqrt{106.25}) = 0.008$$

5. [1] Suppose the sediment density (g/cm) of a randomly selected specimen from a certain region is normally distributed with mean 2.65 and standard deviation .85 (suggested in "Modeling Sediment and Water Column Interactions for Hydrophobic Pollutants," Water Research, 1984: 1169–1174). If a random sample of 25 specimens is selected, what is the probability that the sample average sediment density is at most 3.00?

Answer: **0.980**

$\bar{X}$ is normal with

$$\mu_{\bar{X}} = 2.65, \sigma_{\bar{X}} = \frac{0.85}{\sqrt{25}} = 0.17$$

$$P(\bar{X} < 3) = \text{normalcdf}(-10^{-99}, 3, 2.65, 0.17) = 0.980$$

6. [1] Let $X_1, X_2, \dots, X_{100}$ denote the actual net weights of 100 randomly selected 50-lb bags of fertilizer. If the expected weight of each bag is 50 and the variance is 1, calculate $P(49.9 \leq \bar{X} \leq 50.1)$ (approximately) using the CLT.

Answer: **0.683**

$$\mu_{\bar{X}} = 50, \sigma_{\bar{X}} = \frac{1}{\sqrt{100}} = 0.1$$

$$P(49.9 \leq \bar{X} \leq 50.1) = \text{normalcdf}(49.9, 50.1, 50, 0.1)$$

7. [5] A sample of 26 offshore oil workers took part in a simulated escape exercise, resulting in the accompanying data on time (sec) to complete the escape ("Oxygen Consumption and Ventilation During Escape from an Offshore Platform," Ergonomics, 1997: 281–292):

389 356 359 363 375 424 325 394 402
373 373 370 364 366 364 325 339 393
392 369 374 359 356 403 334 397

- Construct a stem-and-leaf display of the data.
- Compute the sample mean $\bar{x}$ and the sample variance s .
- Find five-number summary (minimum, Q_1 = lower fourth, median, Q_3 = upper fourth, maximum).
- Determine if they are any outliers.
- Draw a boxplot.

I strongly recommend using a calculator, Excel, or other software to answer parts b. and c.

Stem-and-leaf display:

```

32 | 55
33 | 49
34 |
35 | 66 99
36 | 34 46 9
37 | 03 34 5
38 | 9
39 | 23 47
40 | 23
41 |
42 | 4

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b. $\bar{x} = 370.69$ $s = 24.36$

c. minimum = 325, Q_1 = 359, median = 369.5, Q_3 = 392, maximum = 424,

d. $IQR = f_s = 33$, $Q_1 - 1.5 f_s = 309.5$, $Q_3 + 1.5 f_s = 411.5$, **no outliers**

Boxplot



8. [2] Construct a normal probability plot (*make a table as in the class example and plot the points*) for the following sample of observations on coating thickness for low-viscosity paint (“Achieving a Target Value for a Manufacturing Process: A Case Study,” J. of Quality Technology, 1992: 22–26). Would you feel comfortable estimating population mean thickness using a method that assumed a normal population distribution?

Answer: **NO, dots on normal probability plot are not along a straight line**

.83 .88 .88 1.04 1.09 1.12 1.48 1.49 1.59 1.62 1.65 1.71 1.29 1.31 1.76 1.83

<i>i</i>	<i>Percentage =</i> <i>100(i-0.5)/n</i>	<i>z-percentile</i>	<i>Sample</i> <i>Percentile</i>
1	3.125	-1.863	0.83
2	9.375	-1.318	0.88
3	15.625	-1.010	0.88
4	21.875	-0.776	1.04
5	28.125	-0.579	1.09
6	34.375	-0.402	1.12
7	40.625	-0.237	1.48
8	46.875	-0.078	1.49
9	53.125	0.078	1.59
10	59.375	0.237	1.62
11	65.625	0.402	1.65
12	71.875	0.579	1.71
13	78.125	0.776	1.29
14	84.375	1.010	1.31
15	90.625	1.318	1.76
16	96.875	1.863	1.83

