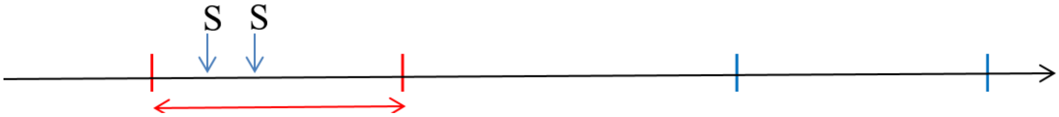
Chapter 4.4 Other Discrete Distributions

1. Poisson Distribution

Distribution of the numbers of rare events



Characteristics of a Poisson Random Variable

1. The experiment consists of counting the number of times x that a certain event occurs during a given period of time (or in a given area, volume, distance, etc).
2. The probability that an event occurs in a given period of time (area, volume,…) is the same for all periods of the same length.
3. The number of events that occur in a given period of time (area, volume,…) is independent of the number that occur in any other mutually exclusive period.
4. The mean (= expected) number of events in a given period is denoted by the Greek letter lambda, λ.

Examples:

- The number of hurricanes in Florida in a month.
- The number of industrial accidents per month at a manufacturing plant
- The number of surface defects (scratches, dents, etc.) found by quality inspectors on a new automobile
- The number of customer arrivals per unit of time at a supermarket checkout counter.
- The number of gas stations in 100 miles section of I-80
- The number of errors per 100 invoices in the accounting records of a company. NOTE: This is, in fact, binomial random variable, but if the total number of invoices is large it can be approximated by a Poisson distribution.

Main difference between Poisson and Binomial distribution: no fixed number of trials. Instead we use the fixed interval of time or space in which the number of successes is recorded.

Poisson Distribution

Probability Distribution is given by the function

Probability Distribution, Mean, and Variance for a Poisson Random Variable*

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad (x = 0, 1, 2, \ldots)$$

$$\mu = \lambda$$

$$\sigma^2 = \lambda$$

$p(x)$ = Probability of x, given:

λ = the mean (expected) number of events in unit $\sigma = \sqrt{\lambda}$

e = 2.71828 . . . (the base of natural logarithm)

x = Number of events **per unit**

Example:

Customers arrive at a rate of 72 per hour. What is the probability of 4 customers arriving in 3 minutes?

Solution:

Lambda= 72 per hour. = 72/60=1.2 per min. = 3.6 per three min. Interval

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$p(4) = \frac{(3.6)^4 e^{-3.6}}{4!} = .1912$$

Example (4.14 p. 213) Suppose the number x of a company's employees who are absent on Mondays has (approximately) a Poisson probability distribution. Furthermore, assume that the average number of Monday absentees is 2.6 (λ = 2.6)

a. Find the mean and standard deviation of x.

b. Use a calculator to find the probability that fewer than two employees are absent on a given Monday.

TI-83: **P(x ≤ k) = poissoncdf(λ,k)**

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poissoncdf(2.6,1) = …

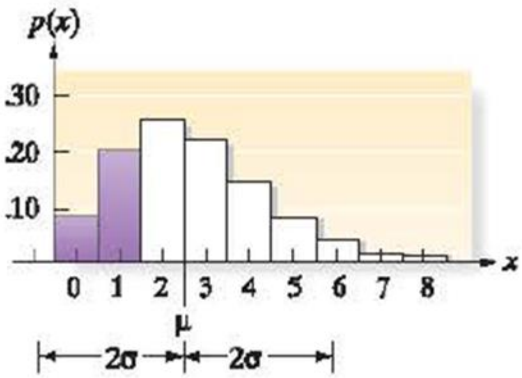
c. Find the probability that more than five employees are absent on a given Monday.

$P(x > 5) = 1 - P(x \leq 5)$ = 1 - poissoncdf(2.6,5) = …

d. Find the probability that exactly five employees are absent on a given Monday.

By hand: $P(x = 5) = (2.6^5 e^{-2.6})/(5!)$ = …
TI-83: **P(*x* = k) = poissonpdf(λ,k)** [2nd→DISTR→B:poissonpdf(….. →ENTER]
$P(x = 5)$ = poissonpdf(2.6,5) = …

e. Graph the distribution of *x*



Example 4.14, p. 213

Exercise (4.62 a-c, p. 217) Assume that *x* is a random variable having a Poisson probability distribution with a mean of 1.5. Find the following probabilities:

a. P (x ≤ 3) = …………………………..

b. P (x ≥ 3) = …………………………..

c. P (x = 3) = …………………………..

Exercise

4.71 Airline fatalities. U.S. airlines average about 4.5 fatalities per month (*Statistical Abstract of the United States: 2012*). Assume the probability distribution for *x,* the number of fatalities per month, can be approximated by a Poisson probability distribution.

a. What is the probability that no fatalities will occur during any given month?

b. What is the probability that one fatality will occur during a month?

c. Find E (x) and the standard deviation of *x*.

2. Hypergeometric Distribution

Hypergeometric distribution is used to model the experiments like selecting from two complementary subsets of given set.

Examples: Draw 5 cards from the deck of cards. Count the diamonds.
Select three persons from a group of 10. Count women.

Unlike in the binomial distribution, the trials in hypergeometric distribution are dependent.

Characteristics of a Hypergeometric Random Variable

1. The experiment consists of randomly drawing *n* elements <u>without replacement</u> from a set of *N* elements, *r* of which are S's (for *success*) and *(N- r)* of which are F's (for *failure*).
2. The hypergeometric random variable *x* is the number of S's in the draw of *n* elements.

Probability Distribution, Mean, and Variance of the Hypergeometric Random Variable

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} \quad [x = \text{Maximum}\,[0, n-(N-r)], \ldots, \text{Minimum}(r, n)]$$

$$\mu = \frac{nr}{N} \qquad \sigma^2 = \frac{r(N-r)n(N-n)}{N^2(N-1)}$$

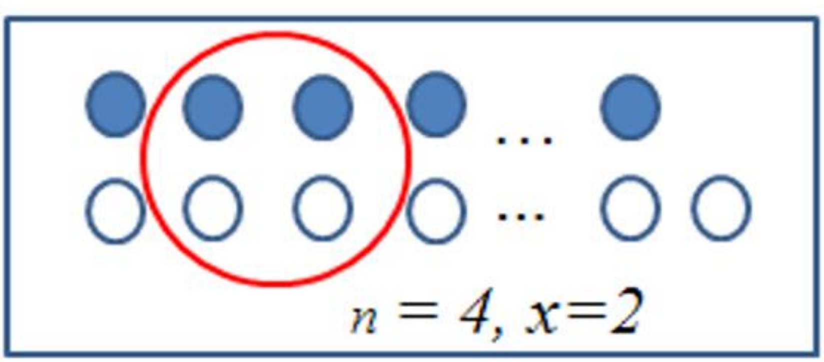
where

N = Total number of elements

r = Number of *S*'s in the *N* elements

n = Number of elements drawn

x = Number of *S*'s drawn in the *n* elements



Example (based on Ex. 4.15, p. 215) Suppose a marketing professor randomly selects three new teaching assistants from a total of ten applicants-six male and four female students. Let *x* be the number of females who are hired.

$N = 10, r = 4, n = 3$

a. Find the mean and standard deviation of x.

b. Find the probability that no females are hired, i.e. that $x = 0$

c. Find the probability that exactly two females are hired, i.e. that $x = 2$

Exercise (4.68 p.217)

4.68 Given that *x* is a hypergeometric random variable with N = 10 , n = 5 , and r = 7 :

a. Display the probability distribution for *x* in tabular form.

b. Compute the mean and variance of *x*.

c. Graph p (x) and locate μ and the interval $\mu \pm 2\sigma$ on the graph.

d. What is the probability that *x* will fall within the interval $\mu \pm 2\sigma$?

4.73. Refer to the investigation of contaminated gun cartridges at a weapons manufacturer, presented in Exercise 4.29 (p. 197). In a sample of 158 cartridges from a certain lot, 36 were found to be contaminated and 122 were “clean.” If you randomly select 5 of these 158 cartridges, what is the probability that all 5 will be “clean”?

4.78 Guilt in decision making. The *Journal of Behavioral Decision Making* (Jan. 2007) published a study of how guilty feelings impact on-the-job decisions. In one experiment, 57 participants were assigned to a guilty state through a reading/writing task. Immediately after the task, the participants were presented with a decision problem where the stated option had predominantly negative features (e.g., spending money on repairing a very old car). Of these 57 participants, 45 chose the stated option. Suppose 10 of the 57 guilty-state participants are selected at random. Define *x* as the number in the sample of 10 who chose the stated option.

a. Find P (x = 5) .
b. Find P (x = 8) .
c. What is the expected value (mean) of *x*?