

## CORRECTION

### LIMIT THEOREMS FOR COUPLED CONTINUOUS TIME RANDOM WALKS

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The converse portion of Theorem 2.2 requires an additional condition, that the probability measure  $\omega$  is such that (2.10) assigns finite measure to sets bounded away from the origin. The argument on page 735 must consider  $B_1$  and  $B_2$  such that at least one is bounded away from zero, not just the case where both are bounded away from zero. The condition on  $\omega$  ensures that the integral on page 735 1.–2 is finite, which is obviously necessary.

The limit process in Theorem 3.4 should read  $A(E(t)-)$ . If  $A(t)$  and  $D(t)$  are dependent, this is a different process than  $A(E(t))$ . To clarify the argument, note that

$$(1) \quad \lim_{h \downarrow 0} \frac{1}{h} P\{A(s) \in M, s < E(t) \leq s + h\} = P\{A(s-) \in M | E(t) = s\} p_t(s),$$

where  $p_t$  is the density of  $E(t)$ , since  $s < E(t)$  in the conditioning event. For an alternative proof, see Theorem 3.6 in Straka and Henry [3]. Theorem 4.1 in [1] gives the density of  $A(E(t)-)$ . Examples 5.2–5.6 in [1] provide governing equations for the CTRW limit process  $M(t) = A(E(t)-)$  in some special cases with simultaneous jumps. Especially, Example 5.5 considers the case where  $Y_i = J_i$  so that  $A(t)$  is a stable subordinator and  $E(t) = \inf\{x > 0: A(x) > t\}$  is its inverse or first passage time process. The beta density for  $A(E(t)-)$  given in that example agrees with the result in Bertoin [2], page 82. Note that here we have  $A(E(t)-) < t$  and  $A(E(t)) > t$  almost surely for any  $t > 0$ , by [2], Chapter III, Theorem 4.

## REFERENCES

- [1] BECKER-KERN, P., MEERSCHAERT, M. M. and SCHEFFLER, H.-P. (2004). Limit theorems for coupled continuous time random walks. *Ann. Probab.* **32** 730–756. [MR2039941](#)
- [2] BERTOIN, J. (1996). *Lévy Processes. Cambridge Tracts in Mathematics* **121**. Cambridge Univ. Press, Cambridge. [MR1406564](#)

- [3] STRAKA, P. and HENRY, B. I. (2011). Lagging and leading coupled continuous time random walks, renewal times and their joint limits. *Stochastic Process. Appl.* **121** 324–336.

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