Your quiz had 5 of the following questions. Note that there was an error in the answers to all the “fill in the blank” questions, so these were not graded. In the solutions, the letter $Z$ always represents a standard normal random variable.

All the questions refer to the following scenario. The length of human pregnancies is normally distributed with mean $\mu$ and standard deviation $\sigma = 16$ days.

1. Suppose that $\mu = 266$. What proportion of pregnancies have a length less than 250 days?

(a) 0.8413  (b) 0.1587  (c) 0.6816  (d) 0.1154  (e) none of the above are correct

**Solution.** Let $X$ stand for the length of a randomly selected woman’s pregnancy. We want $P(X < 250)$, which we can compute by standardizing and using Table B.2.

\[ P(X < 250) = P \left( \frac{X - 266}{16} < \frac{250 - 266}{16} \right) = P(Z < -1) = 0.1587. \]

2. Suppose that $\mu = 266$. Fill in the blank: Exactly 25% of pregnancies are greater than ____________ days in length.

**Solution.** We want the 75th percentile of a normal distribution with mean 266 and standard deviation 16. First find the 75th percentile of a standard normal distribution from Table B.2. Looking for areas near 0.75 in the table, we are led to $z = 0.67$ or $z = 0.68$. I’ll choose $z = 0.67$, although $z = 0.68$ would also be fine. Now multiply by the standard deviation and add the mean to get the answer, $(0.67)(16) + 266 = 276.72$.

3. Suppose that $\mu = 266$. A random sample of $n = 16$ women is included in a study, and the length of their pregnancies is measured. The mean $\bar{X}$ of the 16 lengths is computed. What is the standard deviation of $\bar{X}$?

(a) 16  (b) 1  (c) 4  (d) 66.5  (e) none of the above are correct

**Solution.** The standard deviation of the sample mean is the population standard deviation divided by $\sqrt{n}$. In our case this yields $16/\sqrt{16} = 4$.

4. Suppose that $\mu = 266$. A random sample of $n = 16$ women is included in a study, and the length of their pregnancies is measured. The mean $\bar{X}$ of the 16 lengths is computed. What is the probability that $\bar{X}$ is between 262 and 266?

(a) 0.3413  (b) 0.6826  (c) 0.8413  (d) 0.1587  (e) none of the above are correct

**Solution.** We know (from the “first important fact” from class) that $\bar{X}$ is normally distributed with mean 266 and standard deviation $16/\sqrt{16} = 4$. We want $P(262 < \bar{X} < 266)$, which we’ll compute by standardizing and looking up the answer in Table B.2.

\[
P(262 < \bar{X} < 266) = P \left( \frac{262 - 266}{4} < \frac{\bar{X} - 266}{4} < \frac{266 - 266}{4} \right) = P(-1 < Z < 0) = 0.5000 - 0.1587 = 0.3413.
\]

5. Now suppose that $\mu$ is not known, and that a random sample of $n = 4$ women is included in a study. What is the probability that the mean of their pregnancy lengths, $\bar{X}$, is within 24 of $\mu$?

(a) 0.0013  (b) 0.9987  (c) 0.6816  (d) 0.9974  (e) none of the above are correct
**Solution.** We know (from the “first important fact” from class) that $\bar{X}$ is normally distributed with mean 266 and standard deviation $16/\sqrt{4} = 8$. We want $P(-24 < \bar{X} - \mu < 24)$, which we’ll compute by standardizing and looking up the answer in Table B.2. Since we’ve already got $\bar{X} - \mu$ in the probability statement, standardization involves only dividing by the standard deviation.

\[
P(-24 < \bar{X} - \mu < 24) = P\left(-\frac{24}{8} < \frac{\bar{X} - \mu}{8} < \frac{24}{8}\right) = P(-3 < Z < 3) = 0.9987 - 0.0013 = 0.9974.
\]

6. Suppose that $\mu = 266$. What proportion of pregnancies have a length greater than 250 days?
(a) 0.8413  (b) 0.1587  (c) 0.6816  (d) 0.1154  (e) none of the above are correct

**Solution.** Let $X$ stand for the length of a randomly selected woman’s pregnancy. We want $P(X > 250)$, which we can compute by standardizing and using Table B.2.
\[
P(X > 250) = P\left(\frac{X - 266}{16} > \frac{250 - 266}{16}\right) = P(Z > -1) = 1 - 0.1587 = 0.8413.
\]

7. Suppose that $\mu = 266$. Fill in the blank: Exactly 25% of pregnancies are less than ________ days in length.

**Solution.** We want the 25th percentile of a normal distribution with mean 266 and standard deviation 16. First find the 25th percentile of a standard normal distribution from Table B.2. Looking for areas near 0.25 in the table, we are led to $z = -0.67$ or $z = -0.68$. I’ll choose $z = -0.67$, although $z = -0.68$ would also be fine. Now multiply by the standard deviation and add the mean to get the answer, $(-0.67)(16) + 266 = 255.28$.

8. Suppose that $\mu = 266$. A random sample of $n = 64$ women is included in a study, and the length of their pregnancies is measured. The mean $\bar{X}$ of the 64 lengths is computed. What is the standard deviation of $\bar{X}$?
(a) 16  (b) 2  (c) 4  (d) 66.5  (e) none of the above are correct.

**Solution.** The standard deviation of the sample mean is the population standard deviation divided by $\sqrt{n}$. In our case this yields $16/\sqrt{64} = 2$.

9. Suppose that $\mu = 266$. A random sample of $n = 64$ women is included in a study, and the length of their pregnancies is measured. The mean $\bar{X}$ of the 64 lengths is computed. What is the probability that $\bar{X}$ is between 262 and 266?
(a) 0.9772  (b) 0.6826  (c) 0.9544  (d) 0.0228  (e) none of the above are correct.

**Solution.** We know (from the “first important fact” from class) that $\bar{X}$ is normally distributed with mean 266 and standard deviation $16/\sqrt{64} = 2$. We want $P(262 < \bar{X} < 266)$, which we’ll compute by standardizing and looking up the answer in Table B.2.
\[
P(262 < \bar{X} < 266) = P\left(\frac{262 - 266}{2} < \frac{\bar{X} - 266}{2} < \frac{266 - 266}{2}\right) = P(-2 < Z < 0) = 0.5000 - 0.0228 = 0.4772.
\]
10. Now suppose that $\mu$ is not known, and that a random sample of $n = 4$ women is included in a study. What is the probability that the mean of their pregnancy lengths, $\bar{X}$, is within 8 of $\mu$?
(a) 0.3413  (b) 0.9987  (c) 0.6826  (d) 0.9974  (e) none of the above are correct

Solution. We know (from the “first important fact” from class) that $\bar{X}$ is normally distributed with mean 266 and standard deviation $16/\sqrt{4} = 8$. We want $P(-8 < \bar{X} - \mu < 8)$, which we’ll compute by standardizing and looking up the answer in Table B.2. Since we’ve already got $\bar{X} - \mu$ in the probability statement, standardization involves only dividing by the standard deviation.

$$P(-8 < \bar{X} - \mu < 8) = P\left(\frac{-8}{8} < \frac{\bar{X} - \mu}{8} < \frac{8}{8}\right) = P(-1 < Z < 1) = 0.8413 - 0.1587 = 0.6826.$$ 

11. Suppose that $\mu = 266$. A random sample of $n = 25$ women is included in a study, and the length of their pregnancies is measured. The mean $\bar{X}$ of the 25 lengths is computed. What is the standard deviation of $\bar{X}$?
(a) 16  (b) 1  (c) 0.64  (d) 3.2  (e) none of the above are correct.

Solution. The standard deviation of the sample mean is the population standard deviation divided by $\sqrt{n}$. In our case this yields $16/\sqrt{25} = 3.2$.

12. Suppose that $\mu = 266$. A random sample of $n = 25$ women is included in a study, and the length of their pregnancies is measured. The mean $\bar{X}$ of the 25 lengths is computed. What is the probability that $\bar{X}$ is between 262.8 and 266?
(a) 0.3413  (b) 0.6826  (c) 0.8413  (d) 0.1587  (e) none of the above is true.

Solution. We know (from the “first important fact” from class) that $\bar{X}$ is normally distributed with mean 266 and standard deviation $16/\sqrt{25} = 3.2$. We want $P(262.8 < \bar{X} < 266)$, which we’ll compute by standardizing and looking up the answer in Table B.2.

$$P(262.8 < \bar{X} < 266) = P\left(\frac{262.8 - 266}{3.2} < \frac{\bar{X} - 266}{3.2} < \frac{266 - 266}{3.2}\right) = P(-1 < Z < 0) = 0.5000 - 0.1587 = 0.3413.$$ 

13. Now suppose that $\mu$ is not known, and that a random sample of $n = 4$ women is included in a study. What is the probability that the mean of their pregnancy lengths, $\bar{X}$, is within 4 of $\mu$?
(a) 0.3085  (b) 0.3830  (c) 0.6816  (d) 0.6915  (e) none of the above are correct.

Solution. We know (from the “first important fact” from class) that $\bar{X}$ is normally distributed with mean 266 and standard deviation $16/\sqrt{4} = 8$. We want $P(-4 < \bar{X} - \mu < 4)$, which we’ll compute by standardizing and looking up the answer in Table B.2. Since we’ve already got $\bar{X} - \mu$ in the probability statement, standardization involves only dividing by the standard deviation.

$$P(-4 < \bar{X} - \mu < 4) = P\left(\frac{-4}{8} < \frac{\bar{X} - \mu}{8} < \frac{4}{8}\right) = P(-0.5 < Z < 0.5) = 0.6915 - 0.3085 = 0.3830.$$ 

\[3\]