1. (Page 444, number 5) Compute the area of the region enclosed by the curves \( x = 2y^2 \), \( x = 0 \), and \( y = 3 \).

**Solution:** Let \( f(y) = 2y^2 \) and \( g(y) = 0 \). (The function \( g \) corresponds to the boundary \( x = 0 \).) Then the area is

\[
\int_0^3 (2y^2 - 0) \, dy = 2y^3 \bigg|_0^3 = (54/3) - (0/3) = 18.
\]

2. (Page 445, number 25) Find the volume of the solid generated by revolving the region bounded by the \( x \)-axis, the curve \( y = 3x^4 \), and the lines \( x = 1 \) and \( x = -1 \) about (a) the \( x \)-axis; (b) the \( y \)-axis; (c) the line \( x = 1 \); (d) the line \( y = 3 \).

There is a picture of the region in Figure 1.

**Solution of (a):** A vertical slice of thickness \( \Delta x \) at the point \( x \) is a disk with radius \( 3x^4 \). The volume of such a disk is about \( \pi (3x^4)^2 \Delta x \). So the volume of the solid is

\[
\int_{-1}^{1} \pi (9x^8) \, dx = 2\pi.
\]

**Solution of (b):** The solid is the same as the solid we would obtain by rotating only the portion of the region in the first quadrant about the \( y \)-axis. A horizontal slice of thickness \( \Delta y \) at the point \( y \) is a washer with outside radius \( 1 \) and inside radius \( (y/3)^{1/4} \). (Here I’ve solved for \( x \) in terms of \( y \).) The volume of such a washer is about \( \{\pi(1)^2 - \pi[(y/3)^{1/4}]^2\} \Delta y \). So the volume of the solid is

\[
\pi \int_0^3 [1 - (y/3)^{1/2}] \, dy = \pi.
\]

**Solution to (c):** We’ll first compute the volume of the solid obtained by rotating the portion of the region in the first quadrant about \( x = 1 \). A horizontal slice of thickness \( \Delta y \) at the point \( y \) is a disk with radius \( 1 - (y/3)^{1/4} \). The volume of such a disk is about \( \pi(1 - (y/3)^{1/4})^2 \Delta y \). So the volume of the solid is

\[
\pi \int_0^3 (1 - (y/3)^{1/4})^2 \, dy = \pi/5.
\]

Now consider the portion of the region in the second \( (x < 0, y > 0) \) quadrant. Rotating this about the line \( x = 1 \) gives a washer with outside radius 2 and inside radius \( (1 + (y/3)^{1/4}) \). So the volume of the solid is

\[
\pi \int_0^3 [4 - (1 + (y/3)^{1/4})^2] \, dy = 11\pi/5.
\]
Adding the two volumes gives the volume of the whole solid, namely, $12\pi/5$.

**Solution to (d):** A vertical slice of thickness $\Delta x$ at $x$ has outside radius 3 and inside radius $3 - 3x^4$, so the volume of such a slice is about $\pi [9 - (3 - 3x^4)^2] \Delta x$. The volume of the solid is then

$$\pi \int_{-1}^{1} [9 - (3 - 3x^4)^2] \, dx = 26\pi/5.$$  

3. (Page 446, number 37) Find the length of the curve $y = (5/12)x^{6/5} - (5/8)x^{4/5}$, $1 \leq x \leq 32$.

**Solution:** First compute

$$\frac{dy}{dx} = (1/2)x^{1/5} - (1/2)x^{-1/5}$$

and

$$\left(\frac{dy}{dx}\right)^2 = (1/4)(x^{2/5} - 2 + x^{-2/5}).$$

Now use algebra to write

$$\sqrt{1 + (1/4)(x^{2/5} - 2 + x^{-2/5})} = \sqrt{(1/4)(x^{2/5} + 2 - x^{-2/5})} = \sqrt{(1/4)(x^{1/5} + x^{-1/5})^2} = (1/2)(x^{1/5} + x^{-1/5}).$$

Then integrate this from 1 to 32 to get the length equal to $285/8$.

4. (Page 446, number 49) A rock climber is about to haul up 100 newtons of equipment that has been hanging beneath her on 40 meters of rope that weighs 0.8 newtons per meter. How much work will it take?

**Solution:** The work done lifting the equipment is $\int_{0}^{40} 100 \, dx = 4000$ Joules. For the rope, the work done is

$$\int_{0}^{40} 0.8(40 - x) \, dx = 640 \text{ Joules.}$$

So the total work done is $4000 + 640 = 4640$ Joules.

5. (Page 446, number 52) A force of 200 newtons will stretch a garage door spring 0.8 meters beyond its unstressed length. How far will a 300 newton force stretch the spring? How much work does it take to stretch the spring this far?

**Solution:** Using Hooke’s Law $F = kx$, we obtain $200 = k(0.8)$. Solve for $k$ to get $k = 250$. Again use Hooke’s law with $F = 300$ to get $300 = (250)x$. Solve for $x$ to get $x = 1.2$. So a force of 300 Newtons will stretch the spring 1.2 meters. The work required to stretch the spring this far is

$$\int_{0}^{1.2} 250x \, dx = 180 \text{ Joules.}$$

6. (Page 446, number 53) A reservoir shaped like a right circular cone, point down, 20 feet across the top and 8 feet deep, is full of water. How much work does it take to pump the water to a level 6 feet above the top?

**Solution:** If the bottom of the reservoir is at the origin, then a cross-section of the reservoir looks like Figure 2. The region is bounded by $y = 8$ and $y = (4/5)x$. Now imagine cutting the water horizontally into thin slabs. A slab at $y$ of thickness $\Delta y$ has volume about $\pi ((5/4)y)^2 \Delta y$. Here I’ve used the fact that in the first quadrant, the line $y = (4/5)x$ can be written as $x = (5/4)y$. The force required to lift this slab is about $2(62.4)\pi ((5/4)y)^2 \Delta y = 97.5\pi y^2 \Delta y$. This slab will have to be lifted about $14 - y$ feet. So the work required is

$$\int_{0}^{14} (97.5\pi y^2 (14 - y)) \, dy \approx 418,208.81 \text{ foot pounds.}$$
Figure 2: A cross-section of the reservoir from Problem 6.

7. (Page 548, number 5) Find the derivative of the function \( f(\theta) = \ln(\sin^2(\theta)) \) with respect to \( \theta \).

\[
\begin{align*}
f'(\theta) &= \frac{1}{\sin^2(\theta)} [2 \sin(\theta) \cos(\theta)] = \frac{2 \cos(\theta)}{\sin(\theta)} = 3 \cot(\theta).
\end{align*}
\]

8. (Page 548, number 8) Find the derivative of the function \( f(x) = \log_5(3x - 7) \) with respect to \( x \).

\[
\begin{align*}
\frac{d}{dx} \log_5(3x - 7) &= \frac{d}{dx} \frac{\ln(3x - 7)}{\ln(5)} = \frac{1}{\ln(5)} \frac{1}{3x - 7} (3) = \frac{3}{\ln(5)(3x - 7)}.
\end{align*}
\]

9. (Page 548, number 10) Find the derivative of the function \( f(t) = 9^{2t} \) with respect to \( t \).

\[
\begin{align*}
\frac{d}{dt} 9^{2t} &= \frac{d}{dt} e^{(2t)\ln(9)} = (2t)\ln(9) e^{(2t)\ln(9)} = 9^{2t} (2 \ln(9)).
\end{align*}
\]

10. (Page 548, number 53) Evaluate the following integral:

\[
\int_1^4 \left( \frac{x}{8} - \frac{1}{2x} \right) dx.
\]

\[
\begin{align*}
\int_1^4 \left( \frac{x}{8} - \frac{1}{2x} \right) dx &= (1/2) \int_1^4 \left( \frac{x}{4} - \frac{1}{x} \right) dx = (1/2) \left[ x^2/8 + \ln(x) \right]_1^4 = (15/16) + (1/2) \ln(4) = (15/16) + \ln(2).
\end{align*}
\]

11. (Page 548, number 57) Evaluate the following integral:

\[
\int_0^{\ln(5)} e^{3e^r + 1}^{-3/2} dr.
\]

\[
\text{Solution: Let } u = 3e^r + 1. \text{ Then } du = 3e^r dr; \ u(0) = 4; \text{ and } u(\ln(5)) = 16. \text{ So we need to compute}
\int_4^{16} (1/3)u^{-3/2} du = (1/6).
\]
12. (Page 466, number 58) Evaluate the following integral:

\[ \int_{2}^{4} \frac{1}{x \ln(x)} \, dx. \]

**Solution:** Let \( u = \ln(x) \). Then \( du = \frac{1}{x} \, dx \); \( u(2) = \ln(2) \); and \( u(4) = \ln(4) \). So we need to compute

\[ \int_{\ln(2)}^{\ln(4)} \left( \frac{1}{u} \right) \, du = \ln(u) \bigg|_{\ln(2)}^{\ln(4)} = \ln(4) - \ln(2) = \ln(2). \]

13. (Page 466, number 66) Evaluate the following integral:

\[ \int_{0}^{\pi/12} 6 \tan(3x) \, dx. \]

**Solution:** First write the integral as

\[ \int_{0}^{\pi/12} 6 \left( \frac{\sin(3x)}{\cos(3x)} \right) \, dx. \]

Then let \( u = \cos(3x) \). Then \( du = -3 \sin(3x) \, dx \); \( u(0) = 1 \); and \( u(\pi/12) = (1/\sqrt{2}) \). So we need to compute

\[ \int_{1}^{1/\sqrt{2}} (-2)(1/u) \, du = -2 \left[ \ln(|u|) \right]_{1}^{1/\sqrt{2}} = -2 \ln(1/\sqrt{2}) = 2 \ln(\sqrt{2}) = \ln(2). \]

14. (Page 549, number 79) Solve \( 3^y = 2^{y+1} \) for \( y \).

**Solution:** Take natural logarithms of both sides to get \( y \ln(3) = (y + 1) \ln(2) \). Use algebra to write this as \( y(\ln(3) - \ln(2)) = \ln(2) \). Solve for \( y \) to get \( y = \ln(2)/(\ln(3) - \ln(2)) \).