MTH 133 Lecture 2: Quizzes 1 – 4 Solutions
September 29, 1999 (Vince Melfi)

Quiz 1

1. Find the area between the curves $y = 2 - x^2$ and $y = -x$.

Solution: First determine where the curves intersect by solving $2 - x^2 = -x$ for $x$ to get $x = -1$ and $x = 2$. The area is then

$$\int_{-1}^{2} [2 - x^2 - (-x)] dx = \int_{-1}^{2} [2 - x^2 + x] dx = (2x - x^3/3 + x^2/2)|_{-1}^{2} = 9/2.$$ 

Quiz 2

1. Find the volume of the solid generated when the region bounded by the graphs of $y = x^2$ and $y = 2x$ is revolved about the $x$-axis.

Solution: The functions are plotted in Figure 1. The region is bounded by $x = 0$ and $x = 2$. A vertical slice at $x$ is a disk with outside radius equal to $2x$ and inside radius equal to $x^2$. The area of the disk is $\pi(2x)^2 - \pi(x^2)^2 = \pi(4x^2 - x^4)$. So the volume is

$$\int_{0}^{2} \pi(4x^2 - x^4) dx = \pi(4x^3/3 - x^5/5)|_{0}^{2} = (64/15)\pi.$$
Quiz 3

1. A rectangular swimming pool is 75 feet long, 25 feet wide, and has a uniform depth of 10 feet. If the pool is full, how much work is done in pumping all the water to the level of the top of the pool? Use the fact that water weighs about 62.4 pounds per cubic foot.

Solution: Let the bottom of the pool be situated at \( y = 0 \) and the top at \( y = 10 \). A slice of thickness \( \Delta y \) at \( y \) has weight \((62.4)(25)(75)\Delta y = 117000\Delta y\). The distance this slice has to be moved is about \( 10 - y \) feet. So the work involved in pumping all the water out of the pool is

\[
\int_0^{10} 117000(10 - y) \, dy = 117000\left[10y - \frac{y^2}{2}\right]_0^{10} = 5850000.
\]

Quiz 4

1. Compute the derivative of

\[
\ln \left[ \frac{x + 1}{x + 2} \right].
\]

You need not simplify your answer.

2. Compute

\[
\int_0^\pi \frac{\sin(2t)}{3 - \cos(2t)} \, dt
\]

Solution to Number 1:

\[
\frac{d}{dx} \ln \left[ \frac{x + 1}{x + 2} \right] = \frac{d}{dx} \left[ \ln(x + 1) - \ln(x + 2) \right] = \frac{1}{x + 1} - \frac{1}{x + 2} = \frac{1}{(x + 1)(x + 2)}.
\]

Here’s another method using the formula

\[
\frac{d}{dx} \ln(u) = \frac{1}{u} \frac{du}{dx}
\]

with \( u = \frac{x + 1}{x + 2} \).

\[
\frac{d}{dx} \ln \left[ \frac{x + 1}{x + 2} \right] = \frac{1}{(x + 1)/(x + 2)} \frac{(x + 2)(1) - (x + 1)(1)}{(x + 2)^2} = \frac{1}{(x + 1)(x + 2)}.
\]

Solution to Number 2: The integral is equal to zero. One way to see this is to perform the substitution \( u = 3 - \cos(2t) \). Then \( du = 2\sin(2t) \); \( u(0) = 1 \) and \( u(\pi) = 1 \). Since the limits of integration are equal, the integral is zero.