

Recitation Assignment 10-10-09

Read Chapter 15 (it may help to read through chapter 14 also).

1. Ideally, a statement that there is a 49% chance of winning any particular round of a casino game should have some operational connection to what would happen over 10,000 unrelated plays. Although chance may always surprise us and "defy the odds" around how many wins do we expect over the course of 10,000 plays?

2. *Ideally*, around how many rain days would you expect over 1,000 times a weather forecast says "60% chance of rain today?"

3. Consider two ordinary 6-sided dice, one red and the other green. List the 6x6 array representing all possible outcomes when the two dice are thrown. Denote the number coming up on the red die by R and the number coming up on the green die by G. Using the classical definition of

$$P(\text{event}) = \# \text{ of outcomes favorable} / \# \text{ of outcomes possible}$$

we find $P(R+G = 7) = P((1, 6) \text{ or } (2, 5) \text{ or } \dots (6, 1)) = 6 / 36$. Determine

a. $P(R > G + 2) =$

b. $P(R - G = 4) =$

c. $P(R^2 = 25) =$

4. A box has three one-dollar bills and one five-dollar bill. Jack will draw one of the four bills with equal probability. In the classical equal probability model for this what is $P(\text{Jack gets } 5)$?

5. Same as (4) except that following Jack's draw Jill will draw a bill with equal probability from the three bills then remaining (i.e. Jack keeps the bill he draws and Jill gets to draw from the three then remaining). At the *intuitive level*:

a. What would you say is the probability Jill gets 5?

b. What would you say is the probability Jill gets 5 if you are informed that Jack got 5?

c. What would you say is the probability Jill gets 5 if you are informed that Jack got a 1?

6. A classical model for answering questions in #5 enumerates all ways the draws of Jack and Jill can proceed. Denote the four bills by 1a, 1b, 1c, 5 and list all possible draws of two (Jack's draw followed by Jill's draw).

7. In your list #6:

- a. What fraction of the possible outcomes are ones in which Jill gets 5?
 - b. What fraction of the possible outcomes in which Jack gets 5 are ones in which Jill gets 5?
 - c. What fraction of the possible outcomes in which Jack gets a 1 are ones in which Jill gets 5?
8. How do your answers to #7 compare with your intuitive answers in #5? Are you convinced of the relevance of your answers to #7 to the actual experiment of performing the Janck & Jill experiment many times?
9. A box contains 2 red balls, 3 Green balls and 1 Yellow ball. This will be written [2R 3G 1Y]. Two draws will be made from the box with equal probability and *without*-replacement. Give:
- a. The probability the first ball is Red, denoted $P(R1)$.
 - b. The probability the first ball is Red or Green, denoted $P(R1 \cup G1)$.
 - c. The *intuitive* probability the second ball is Red, denoted $P(R2)$.

10. Refer to #9c. List the contents of the box as

[ra rb ga gb gc y]

and enumerate all possible outcomes of two draws from the box without replacement. There are $6 \times 5 = 30$ of these possible outcomes. Using this enumeration find the classical probability $P(R2) = \text{ratio of \#favorable outcomes to } \mathbf{30}$. How does it compare with your answer 9c?

11. Refer to the box [ra rb ga gb gc y].

- a. If we are informed that the first draw is Red, what is the probability we then give for the second draw being Red?
- b. If we are informed that the first draw is not Red, what is the probability we then give for the second draw being Red?
- c. If we are informed that the first draw is Yellow, what is the probability we then give for the second draw being Red?
- d. If we are informed that the first draw is Green, what is the probability we then give for the second draw being Red?
- e. If we are informed that the first draw is Yellow, what is the probability we then give for the second draw being Yellow?

12. Refer to #10 but now consider equal-probability draws **with**-replacement from the box [ra rb ga gb gc y]. To enumerate all possible draws of two from the six balls now requires **36** pairs, including repeats such as ra1, ra2 (ra on each of draws one and two). Enumerate them all. Using this enumeration find the classical probabilit-

ity $P(R2) = \text{ratio of \#favorable outcomes to } 36$. How does it compare with your answer 9c?

13. Refer to #12. Determine

- a. If we are informed that the first draw is Red, what is the probability we then give for the second draw being Red?
- b. If we are informed that the first draw is not Red, what is the probability we then give for the second draw being Red?
- c. If we are informed that the first draw is Yellow, what is the probability we then give for the second draw being Red?
- d. If we are informed that the first draw is Green, what is the probability we then give for the second draw being Red?
- e. If we are informed that the first draw is Yellow, what is the probability we then give for the second draw being Yellow?

With-replacement draws are said to be *statistically independent* since nothing we learning about outcomes to some of the draws changes our probability for unseen outcomes to other draws.

14. In your readings pay attention to

Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Multiplication rule: $P(A) P(B \text{ if } A) = P(A \cap B)$.

Conditional probability: $P(B \text{ if } A) = P(A \cap B) / P(A)$

which we take up 11-19-09. In classical probability models these rules refer to ratios of favorable cases to total cases.

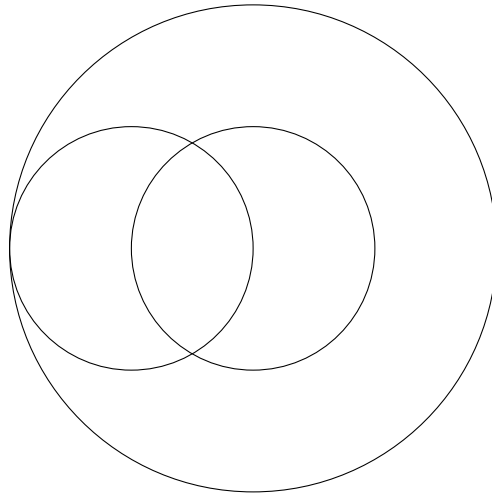
THESE RULES ARE SIMPLE PROPERTIES OF RELATIVE COUNTS.

The addition rule in the classical model simply says

$$\frac{\# \text{ in } (A \cup B)}{\# \text{ total}} = \frac{\# \text{ in } A}{\# \text{ total}} + \frac{\# \text{ in } B}{\# \text{ total}} - \frac{\# \text{ in } (A \cap B)}{\# \text{ total}}$$

regions 1+2+3+4 2+3 3+4 2

a. Number the four non-overlapping regions of a Venn Diagram for A, B identifying events A, B, their union $A \cup B$ and their intersection $A \cap B$ as indicated above.



b. If $P(A) = 0.7$, $P(B) = 0.6$ and $P(A \cap B) = 0.4$ fill out the correct probabilities of all four events shown in the Venn diagram (use the fact that the four probabilities must total one).

c. Refer to (b). Determine $P(B \text{ if } A)$, written $P(B \mid A)$.

d. Refer to (b). Determine $P(A \text{ if } B)$, written $P(A \mid B)$.

e. Refer to (b). Determine $P(A \cup B)$.

f. Refer to (b). Verify that $P(A \cup B) =$ combined probability of regions 2, 3, 4 = $P(A) + P(B) - P(A \cap B)$.