Solutions to Recitation 10/10

1. $0.49(10000) = 4900$

2. $0.60(1000) = 600$

3. I will use the notation $(\#, \#)$.

$(\underset{\text{red die}}{\#}, \underset{\text{green die}}{\#})$

all possible outcomes $= \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

Note: this is with replacement

There are 36 outcomes in total from counting the above.

- $P(R+G=7) = 6/36 = 1/6$ because for $R+G=7$, we can have the following outcomes:
  $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$
  which are 6, and we divide 6 by the total number of outcomes to find the probability.

a) $P(R > G+2) = 6/36 = 1/6$
Outcomes: $(4,1), (5,1), (5,2), (6,1), (6,2), (6,3)$

b) $P(R-G=4) = 2/36 = 1/18$
Outcomes: $(5,1), (6,2)$

c) $P(R^2 = 25) = 6/36 = 1/6$
Outcomes: $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$

4\. P(Jack gets/draws 5 dollar bill) $= \frac{1}{4}$
Outcome : \$5 bill
There are 4 bills in total.

5\. a) Depends on what Jack got, since he draws first.
b) 0
c) $1/3$

6\. all possible outcomes $= \{(1a, 1b), (1a, 1c), (1a, 5)$
$(1b, 1a), (1b, 1c), (1b, 5)$
$(1c, 1a), (1c, 1b), (1c, 5)$
$(5, 1a), (5, 1b), (5, 1c)\}$

Note: this is without replacement

7\. a) $3/12 = 1/4$
b) $0/3 = 0$
c) $3/9 = 1/3$

note: in (b) we restrict ourselves to looking at just the outcomes in which Jack gets 5
smaller group : $(5, 1a), (5, 1b), (5, 1c)$
and we determine how many of these 3 outcomes are ones in which Jill gets 5 (this has to do with conditional probability)

note: in (c) we restrict ourselves to looking at the outcomes : $(1a, 1b), (1a, 1c), (1a, 5)$
$(1b, 1a), (1b, 1c), (1b, 5)$
$(1c, 1a), (1c, 1b), (1c, 5)$

8. Parts (b) and (c) in Q5 and Q7 are the same.

9. a) $P(\text{first ball is red}) = 2/6 = 1/3$

b) $P(\text{first ball is red OR green}) = 5/6$

c) Depends on what color ball was drawn on the first draw.

* answer $= \frac{1}{3}$ (explanation on last page)

10. I'm going to label the contents $[\underbrace{ra, rb}_{\text{2 reds}}, \underbrace{ga, gb, gc}_{\text{3 greens}}, \underbrace{y}_{\text{1 yellow}}]$.

$$\text{all possible outcomes} = \left\{ \begin{array}{l} (ra, rb), (ra, ga), (ra, gb), (ra, gc), (ra, y) \\ (rb, ra), (rb, ga), (rb, gb), (rb, gc), (rb, y) \\ (ga, ra), (ga, rb), (ga, gb), (ga, gc), (ga, y) \\ (gb, ra), (gb, rb), (gb, ga), (gb, gc), (gb, y) \\ (gc, ra), (gc, rb), (gc, ga), (gc, gb), (gc, y) \\ (y, ra), (y, rb), (y, ga), (y, gb), (y, gc) \end{array} \right\}$$

<u>Note</u>: this is without replacement

$P(\text{Second draw is red}) = 10/30 = 1/3$ (Same as Q9(c))

Outcomes: (ra, rb), (rb, ra), (ga, ra), (ga, rb), (gb, ra)
(gb, rb), (gc, ra), (gc, rb) (y, ra), (y, rb)

The setup is the same as in Q9(c) so we expect to get the same answer.

11. a) restricted to looking at outcomes:

(ra, rb), (ra, ga), (ra, gb), (ra, gc), (ra, y)
(rb, ra), (rb, ga), (rb, gb), (rb, gc), (rb, y)

\# of outcomes with 2^{nd} draw being red = 2

$P(2^{nd} \text{ draw is red if } 1^{st} \text{ draw was red})$

$= P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw red}) = \frac{2}{10} = \frac{1}{5}$

b) $P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw not red}) = 8/20 = 2/5$

c) $P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw yellow}) = 2/5$

d) $P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw green}) = 6/15 = 2/5$

e) $P(2^{nd} \text{ draw yellow} \mid 1^{st} \text{ draw yellow}) = 0/5 = 0$

12. all possible outcomes = { (ra, ra), (ra, rb), (ra, ga), (ra, gb), (ra, gc), (ra, y)
(rb, ra), (rb, rb), (rb, ga), (rb, gb), (rb, gc), (rb, y)
(ga, ra), (ga, rb), (ga, ga), (ga, gb), (ga, gc), (ga, y)
(gb, ra), (gb, rb), (gb, ga), (gb, gb), (gb, gc), (gb, y)
(gc, ra), (gc, rb), (gc, ga), (gc, gb), (gc, gc), (gc, y)
(y, ra), (y, rb), (y, ga), (y, gb), (y, gc), (y, y) }

Note: this is without replacement

$P(2^{nd} \text{ draw is red}) = 12/36 = 1/3$

outcomes: (ra, ra), (ra, rb), (rb, ra), (rb, rb), (ga, ra), (ga, rb)
(gb, ra), (gb, rb), (gc, ra), (gc, rb), (y, ra), (y, rb)

We get the same answer as Q9(c), but the setup is different. This will not always be true.

13. a) $P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw red}) = 4/12 = 1/3$

b) $P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw not red}) = 8/24 = 1/3$

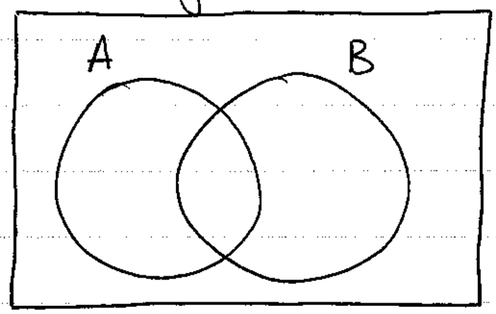
c) $P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw yellow}) = 2/6 = 1/3$

d) $P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw green}) = 6/18 = 1/3$

e) $P(2^{nd} \text{ draw yellow} \mid 1^{st} \text{ draw yellow}) = 1/6$

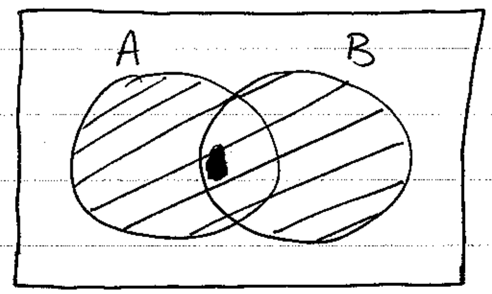
14. Venn Diagrams

a)

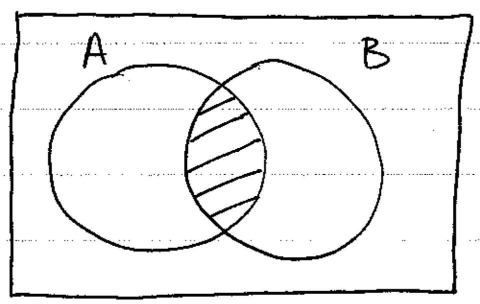


The smaller circles represent events A and B respectively. We can draw events A and B inside a box or a circle, both are okay (A and B drawn inside a circle on worksheet)

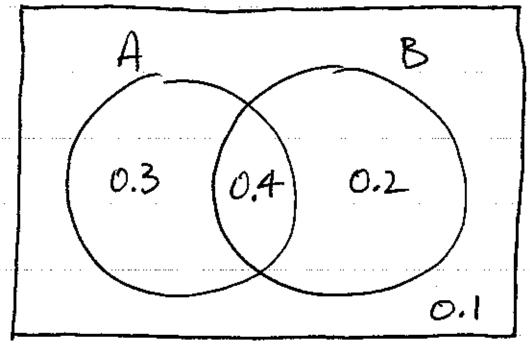
$A \cup B$: reads "A union B" or "A OR B"



$A \cap B$: reads "A intersect B" or "A AND B"



b)



All non-overlapping areas <u>always</u> add up to 1.

c) $P(B|A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.4}{0.7} = \dfrac{4}{7}$

We restrict ourselves to the circle A, and find the portion of B relative to this restricted area.

d) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.4}{0.6} = \dfrac{2}{3}$

We restrict ourselves to the circle B, and find the percentage of A relative to this restricted area.

e) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.6 - 0.4$

$= 0.9$

f) Same as (e).

9. (c) There are 2 cases to consider.

<u>Case 1</u>

The 1^{st} ball drawn was red.

$P(2^{nd}$ ball is red$) =$

chance that 1^{st} ball is red $\times$

chance that 2^{nd} ball is red

$= \frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$ (1 red left on 2^{nd} draw)

Case 2

The first ball drawn is not red.

$P(2^{nd}$ ball is red$)$ = chance that 1^{st} ball is not red × chance that 2^{nd} ball is red

$= \frac{4}{6} \times \frac{2}{5}$

$= \frac{8}{30}$ (2 red balls left in 2^{nd} draw)

$P(2^{nd}$ ball is red$)$ = case 1 + case 2 (all possible cases added together)

$= \frac{2}{30} + \frac{8}{30} = \frac{10}{30}$

$= \frac{1}{3}$

* You will better understand this question if you look at it last.