

Exercises due at the close of recitation Tuesday, 3-24-09. **Circle Answers!**

**1.  $P(A) = 0.7$ ,  $P(B) = 0.4$ ,  $P(AB) = 0.2$ .**

a. Fill out a complete Venn diagram, label each of four pieces (events) and give their probabilities (must total one).

b. Determine  $P(B | A)$  and  $P(B | \text{not } A)$ . Make a complete Tree Diagram with initial A, not A branch and downstream branchings B, not B. Label EVERYTHING and completely place all probabilities in the tree. Especially, include the terminal events AB, A(not B) etc, and their probabilities (which are really the Venn Diagram).

c. From the Venn Diagram, find  $P(B)$  (total probability) and  $P(A | B)$  (Bayes).

d. From the Tree Diagram determine  $P(A | B)$  (Bayes).

**2. We are given  $P(OIL) = 0.2$ ,  $P(+ | OIL) = 0.9$ ,  $P(+ | \text{no OIL}) = 0.3$ .**

a. Fill out a complete Tree Diagram. Label everything.

b. From the tree determine  $P(+)$  and  $P(\text{OIL} \mid +)$ .

c. Cost to test = 40, cost to drill = 70, gross from finding oil = 500. Calculate  $E(\text{NET return})$  from the policy "just drill."

d. In your tree (a) fill in the net returns for the policy "test and only drill if the test is +."

e. Calculate  $E(\text{NET return from policy (d)})$ .

f. Compare (c) with (e). Based on expectations, does it seem worthwhile to test? Does either policy have positive expected return?

**4.  $P(A) = 0.4$ ,  $P(B) = 0.5$ ,  $P(AB) = 0.22$ .**

a. Use addition rule to determine  $P(A \cup B)$ .

b. Use definition to determine  $P(B \mid A)$ .

c. Are A, B independent of each other? Show reasoning!

**5.  $P(A) = 0.5$ ,  $P(\bar{B}) = 0.2$ ,  $P(B | A) = 0.8$ .**

- Give  $P(AB)$ .
- Are A, B independent?
- Fill out a complete Venn Diagram.

**6. Let X denote r.v. "number drawn at random from {2 4 4 6 8 12}. Let Y denote r.v. "number drawn at random from {2 2 2 6}.**

- $E X =$
- $\text{Var } X =$                                        $\text{sd } X =$
- $E Y =$                                        $\text{Var } Y =$
- $E(4 X - Y + 3) =$
- IF X, Y ARE INDEPENDENT,  $\text{Var}(4 X - Y + 3) =$   
(and the name associated with this calculation is                                      )

**7. A game of chance has return  $X$  with  $E X = -\$0.57$  and  $\text{Var } X = \$4.88$ . Let  $T =$  total of 100 independent plays  $T = X_1 + X_2 + \dots + X_{100}$ .**

a.  $E T =$

b.  $\text{Var } T =$                        $\text{sd } T =$

c. The distribution of  $T$  (all of its possible values  $t$ , together with their probabilities) would seem to be a complicated entity. Indeed it is! But a famous theorem (the Central Limit Theorem denoted CLT) says this distribution (under some specified conditions) is approximated by a normal distribution having the mean (a) and sd (b). Sketch this bell curve approximation of the distribution of  $T$  and place the mean and sd appropriately in your sketch. Reflect on the fact that we know little about the actual game, only the expectation and standard deviation of its return on each play  $X$ . From this we are able to simply describe the economic outcome (total) from many plays.

d. Give an interval range within which  $T$  falls with probability around 0.68.

e. Do the bell curve for the total of 10,000 independent plays, such as a casino might have in a single night.

f. Comparing  $E T$  with  $\text{sd } T$  notice that  $T$  is likely to be very close to  $E T$  in the sense that  $\text{sd } T / E T$  is not very large. Calculate  $\text{sd } T / E T$ .

g. Work out the ratio  $\text{sd } T / E T$  for 1 Million plays. The casino's take is practically a sure thing!