Exercises due at the close of recitation Tuesday, 3-24-09. **Circle Answers!**

1. P(A) = 0.7, P(B) = 0.4, P(AB) = 0.2.

a. Fill out a complete Venn diagram, label each of four pieces (events) and give their probabilities (must total one). **$P(A\ B^C) = P(A) - P(AB)$**

b. Determine P(B | A) and P(B | not A). Make a complete Tree Diagram with initial A, not A **(i.e. A^C)** branch and downstream branchings B, not B. Label EVERYTHING and completely place all probabilities in the tree. Especially, include the terminal events AB, A(not B) etc, and their probabilities (which are really the Venn Diagram). **P(B | A) = P(AB) / P(A)**

$P(B \mid A^C) = P(A^C B) / P(A^C)$

c. From the Venn Diagram, find P(B) (total probability) and P(A | B) (Bayes).

$P(B) = P(AB) + P(A^C B)$
P(A | B) = P(AB) / P(B)

d. From the Tree Diagram determine P(A | B) (Bayes).

2. We are given P(OIL) = 0.2, P(+ | OIL) = 0.9, P(+ | no OIL) = 0.3.

a. Fill out a complete Tree Diagram. Label everything. **P(AB) = P(A) P(B | A) then as #1a.**

b. From the tree determine P(+) and P(OIL | +). **P(+) = P(OIL+)+P(no OIL +)**
P(OIL | +) = P(OIL+) / P(+)

c. Cost to test = 40, cost to drill = 70, gross from finding oil = 500. Calculate E(NET return) from the policy "just drill."

E(NET return) = $\sum$ x p(x) = (-70 + 500) 0.2 + (   ) (   )

d. In your tree (a) fill in the net returns for the policy "test and only drill if the test is +."
At tree endpoint OIL+ net return times probbility is (-40). Do the other three cases.

e. Calculate E(NET return from policy (d)). **Total the four values x p(x) from (d).**

f. Compare (c) with (e). Based on expectations, does it seem worthwhile to test? Does either policy have positive expected return?

4. PA) = 0.4, P(B) = 0.5, P(AB) = 0.22.

a. Use addition rule to determine P(A⋃B). **P(A⋃B) = P(A) + P(B) - P(AB) always.**

b. Use definition to determine P(B | A). **P(B | A) = P(AB) / P(A).**

c. Are A, B independent of each other? Show reasoning! **Does P(AB) = P(A)P(B)?**

5. P(A) = 0.5, P(B) = 0.2, P(B | A) = 0.8.

a. Give P(AB). **P(AB) = P(A) P(B | A) always (excepting P(A) = 0 in which case P(B | A) is defined to be P(B)).**

b. Are A, B independent? **Is P(B) = P(B | A)?**

c. Fill out a complete Venn Diagram.

6. Let X denote r.v. "number drawn at random from {2 4 4 6 8 12}. Let Y denote r.v. "number drawn at random from {2 2 2 6}.

a. E X = **2(1/6) + 4(2/6) + 8(1/6) + 12(1/6) (or just (2 + 4 + 4+ 6 + 8 + 12) / 6)**

b. Var X = $\mathbf{E\,X^2 - (E\,X)^2}$ sd X = $\mathbf{\sqrt{Var\,X}}$

$\mathbf{E\,X^2 = \sum x^2\,p(x)}$

c. E Y = Var Y =

d. E(4 X - Y + 3) =(addition rule of E)

e. IF X, Y ARE INDEPENDENT, Var(4 X - Y + 3) = **addition rule, Var of independent r.v.** (and the name associated with this calculation is **famous Greek**)

7. A game of chance has return X with E X = -$0.57 and Var X = $4.88. Let T = total of 100 independent plays T = X1 + X2 +..... + X100.

a. E T = **E X1 + E X2 +..... + E X100 =**

b. Var T $\overset{\textbf{if independent } r.v.}{=}$ **Var X1 + ... + Var X100 =** sd T =

c. The distribution of T (all of its possible values t, together with their probabilities) would seem to be a complicated entity. Indeed it is! But a famous theorem (the Central Limit Theorem denoted CLT) says this distribution (under some specified conditions) is approximated by a normal distribution having the mean (a) and sd (b). Sketch this bell curve approximation of the distribution of T and place the mean and sd appropriately in your sketch. Reflect on the fact that we know little about the actual game, only the expectation and standard deviation of its return on each play X. From this we are able to simply describe the economic outcome (total) from many plays.

d. Give an interval range within which T falls with probability around 0.68.

e. Do the bell curve for the total of 10,000 independent plays, such as a casino might have in a single night.

f. Comparing E T with sd T notice that T is likely to be very close to E T in the sense that sd T / ET is not very large. Calculate sd T / E T.

g. Work out the ratio sd T / E T for 1 Million plays. The casino's take is practically a sure thing!