STT200 3-30-09a.

Prep for EXAM 3 (TOMORROW IS "EXAM 3 DAY")

SHOW ENOUGH WORK?

COUNTS

COUNTS

CAN BE COUNTS GEN'L SUM OF RETURNS

BEGIN WITH BINOMIAL + POISSON + TOTALS $T = X_1 + \cdots + X_n$

PARTICULARLY EMPHASIZING NORMAL APPROX.

THINK BINOMIAL $n \sim \infty$ $p \sim 0$

POISSON APPLIES TO COUNTS OF RARE EVENTS.

AVG 16 ERUPTIONS/MONTH

IF COUNT X IN GIVEN MONTH IS POISSON

THEN P(GET 14 IN GIVEN MONTH) $= P(X=14) = p(14)$

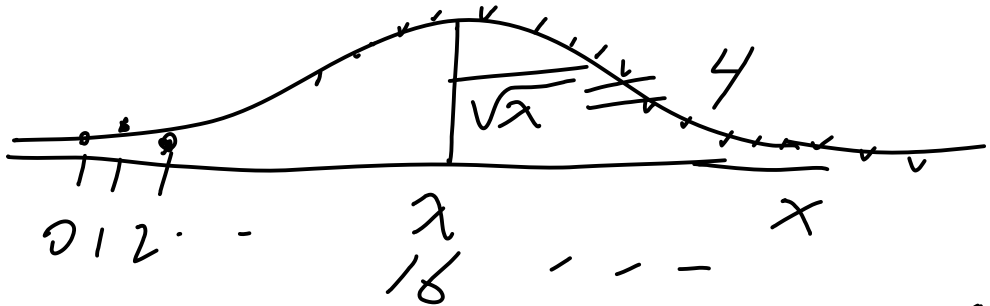
$\underset{\text{FOR POISSON}}{=} e^{-16} \frac{16^{14}}{14!}$

$e^{-\lambda} \lambda^x / x!$

$x = 0, 1, \ldots$ ad inf

FOR $\lambda \geq 3$ NORMAL APPROX APPLIES $\lambda = EX$

POISSON $\approx$



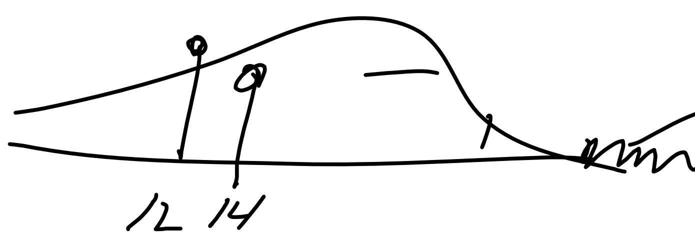
$P(X \text{ IN } 16 \pm 4) \underset{\text{NAIVE}}{\approx} .68$

$16 \pm 8 \approx .95$

NAIVE = W/O USING "CONTINUITY CORRECTION"

NOT NOW

eg IN A MONTH, IT IS VERY UNLIKELY WE'D SEE $X > 16+12$ (3 SD OUT)



SMALL PR.

BINOMIAL n, p : $EX = np$, $Var = np(1-p) = npq$

TYPICAL APP: DELIVERY OF 12,000 PARTS. $q = 1-p$

FPC: $\sqrt{\frac{N-n}{N-1}} \sim 1$

SUPPOSE 20% ARE FAULTY (WE DON'T KNOW THIS).

$p = .2$ SAMPLE OF $n = 36$.

X = # FAULTY PARTS IN SAMPLE. POSS VALUE $x = 0, 1, \cdots, 36$

$P(X=0) = \underbrace{GGG\cdots G}_{36} \underset{\text{REPL}}{\overset{\sim \text{WITH}}{=}} \underbrace{.8 \; .8 \; .8 \cdots .8}_{} = .8^{36}$ (NO FAULTY)

$P(X=1) = P(FG\cdots G) + P(GFG\cdots G) + \cdots + P(G\cdots GF)$

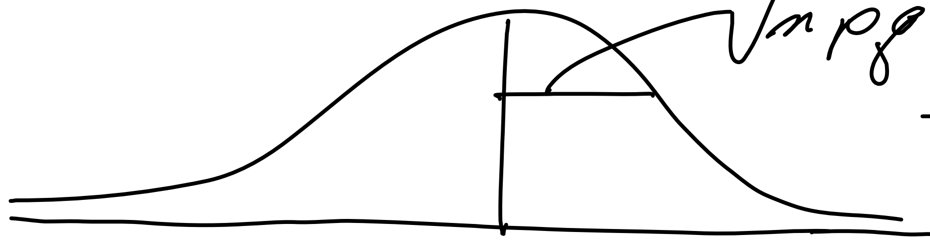
$= 36 \; .2^{1} \; .8^{35}$ $n-1$

"EXPLAINS" BIN $p(x) = {}_nC_x \, p^x q^{n-x}$ ${}_nC_x = \frac{n!}{x!(n-x)!}$

GREATER IMPORTANCE IS NORMAL APPROX

BIN ~

$n \to \infty$
$p \not\to 0$
$q \not\to 0$



X

$\sqrt{npq} = \sqrt{36 \cdot .2 \cdot .8}$
$= 6(.4)$
$= 2.4$

$EX = np = 7.2$

TRY 36 TIMES   PR FAULTY

$36(.2) = 7.2$ (FIX)

TOTALS   CASINO GAME   RANDOM X KNOW $EX = 6.3$ (SAY)
$\mathrm{Var}\, X = 8.4$ μ σ^2

MANY INDEP PLAYS

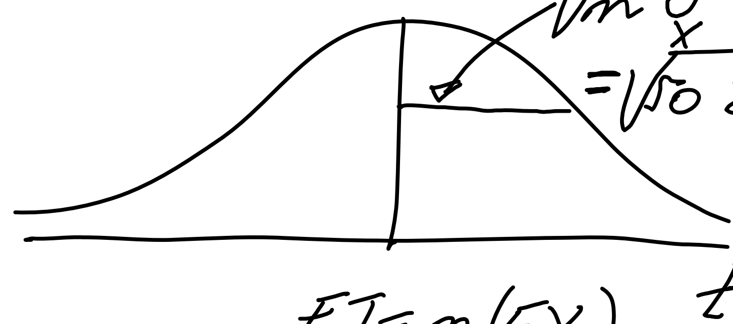
??? $\to$ $T = X_1 + \cdots + X_n$ TOTAL OF n RANDOM PLAYS.

CLT ← THEOREM

LIMIT

CENTRAL

$\sim$ DIST OF



$\sqrt{n\sigma_X^2} = \sqrt{50}\ 8.4$

SAY $n = 50$

$ET = n(EX)$

$= 50(6.3)$

GO BACK TO PROB^Y + VENN + TREE

$\{3R\ 2G\ 6B\}$ $\{4R\ 7G\ 9B\}$

I II

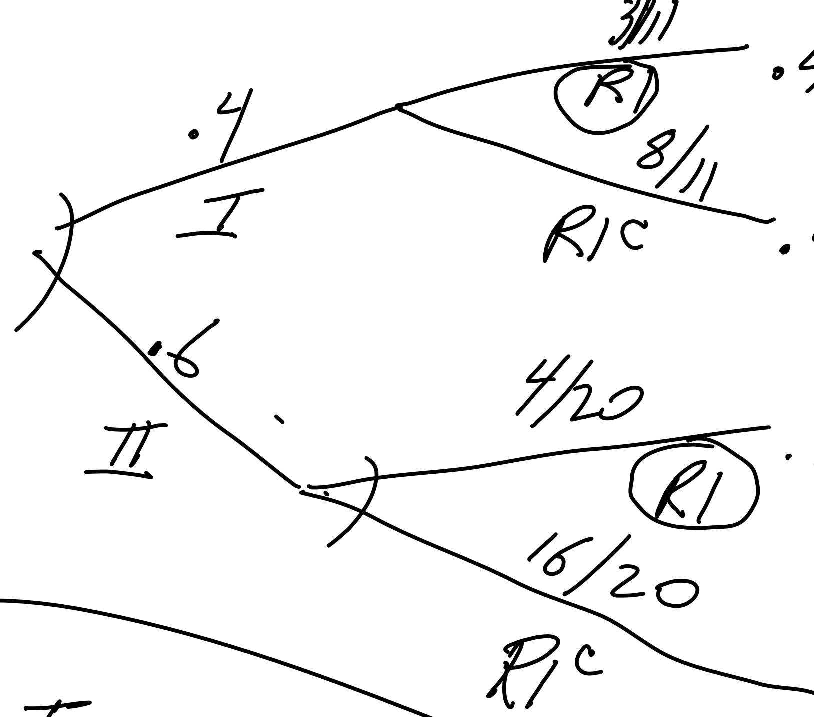
$P(I) = .4$ $P(II) = .6$

PICK BOX — DRAW FROM THAT BOX W/O REPL.

$$P(R1) = P(I)P(R1|I) + P(II)P(R1|II)$$

$$.4\ \ 3/11 + .6\ \ 4/20$$

TREE



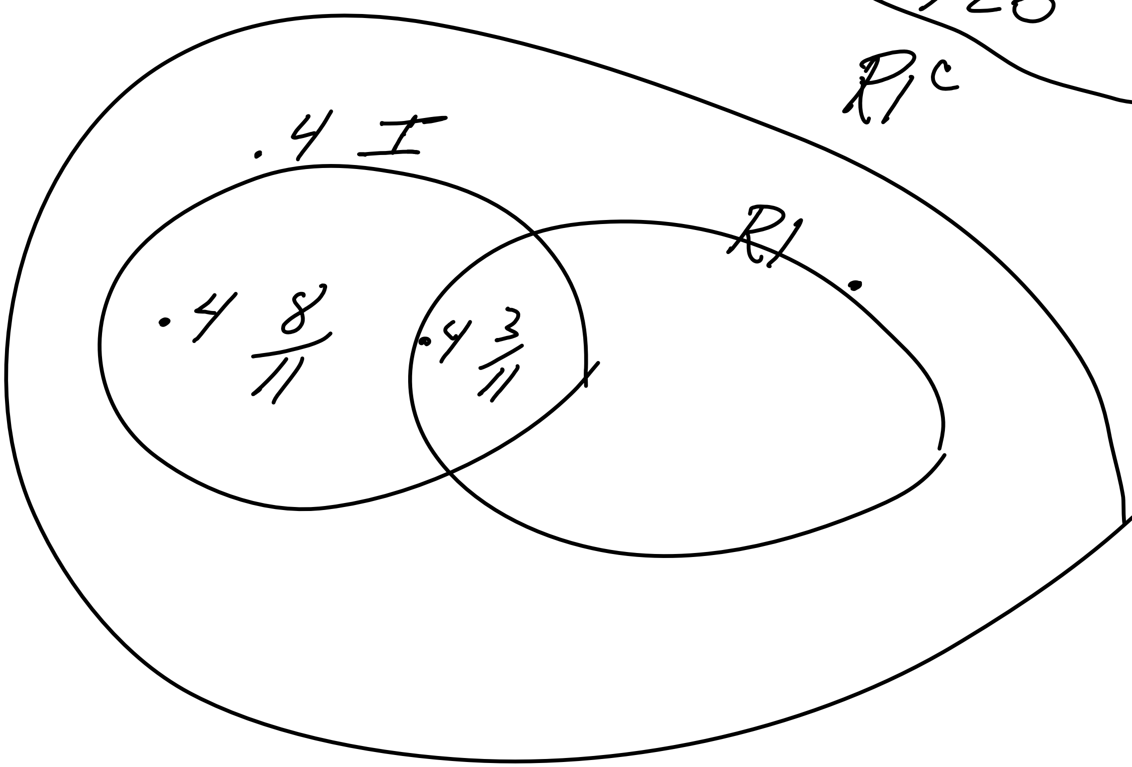
$.4\ \frac{3}{11} = P(I\ RI)$

$.4\ \frac{8}{11} = P(I\ RI^c)$

$.6\ \frac{4}{20} = P(II\ RI)$

$.6\ \frac{16}{20} = P(II\ RI^c)$

VENN



$P(RI) = .4\ \frac{3}{11} + .6\ \frac{4}{20}$

"Some other sets"

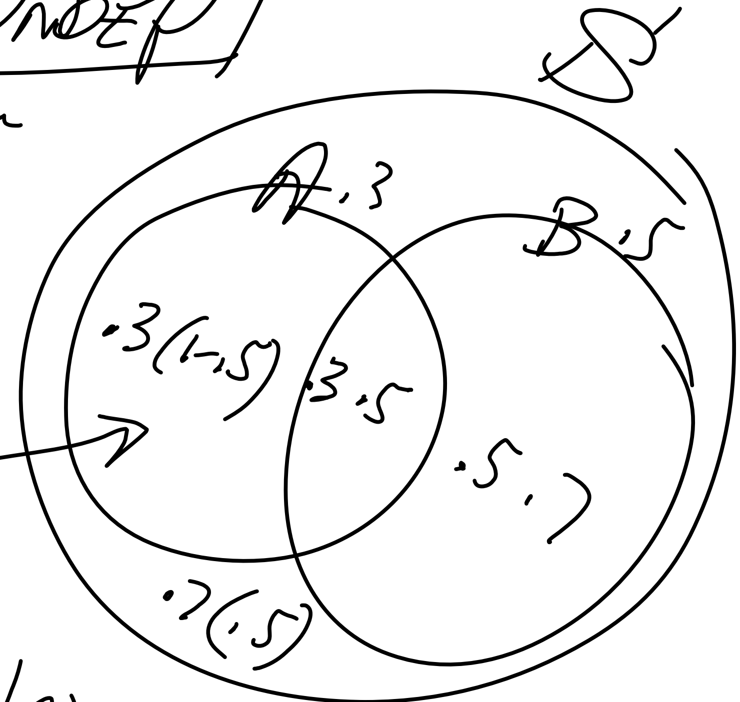
1. $P(A) = .3 \quad P(B) = .5$ $\boxed{\text{INDEP}}$

$P(AB) = .3(.5) = .15$

$P(B|\cancel{A})$

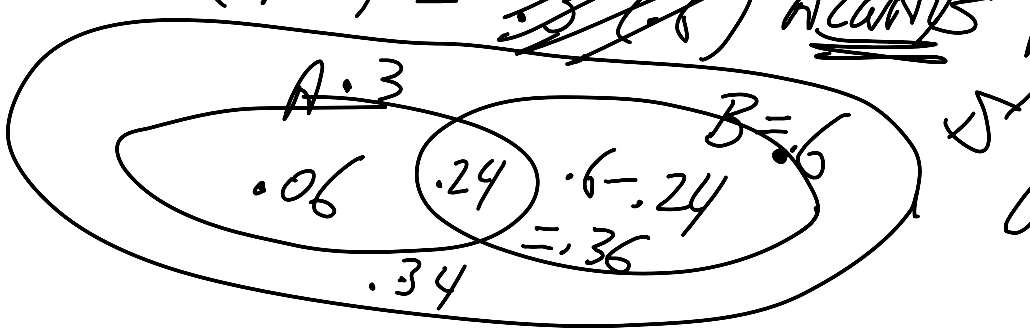
INDEP = NO INFO

$P(AB^c) = P(A)\,P(B^c)$



2. $P(A) = .3,\ P(B) = .6,\ P(B|A) = .8$

$P(AB) = \cancel{.3(.6)}$ ALWAYS $P(A)\,P(B|A) = .3(.8)$
$= .24$



OK

BOX $\{R\ 3G\}$ W/O REPL

$P(R1) = 6/9$

$P(R2 | R1) \underset{\text{SEE IT}}{=} 5/8$

IF ("COND'L ON")

DRAWS FROM $\{5R\ 3G\}$

THEN $P(R1\,R2) \underset{\text{MULT}}{=} P(R1)\,P(R2|R1)$

$= 6/9 \cdot 5/8$

ALSO

$P(R1^c R2) = P(R1^c)\,P(R2 | R1^c)$

$\{6R\ 2G\}$

$= 3/9 \cdot 6/8$

$P(R2)$

$= P(R1\,R2) + P(R1^c R2)$

TOT PR RULE $= 6/9 \cdot 5/8 + 3/9 \cdot 6/8$

$= \dfrac{6 \cdot 5 + 3 \cdot 6}{9 \cdot 8} = \dfrac{6 \cdot \cancel{8}}{9 \cdot \cancel{8}}$

SAME AS $P(R1)$

COR: $P(R2 | R1) \stackrel{\text{DEF}}{=} P(R1\,R2) / P(R1) = \dfrac{\cancel{6/9} \cdot 5/8}{\cancel{6/9}} = 5/8$