STT 200 3-30-09b

PREP FOR EXAM 3 15 QUESTIONS

SHOW ENOUGH WORK

DO NOT REDUCE

eg $3/8$ $4/5$ + + - - ..

LEAVE IT

[BINOMIAL, POISSON, TOTAL $T = X_1 + \cdots + X_n$ many plays of game

IN RELATIONSHIP WITH NORMAL APPROX.

CENTRAL LIMIT THEOREM. = [BELL APPROX OF DIST OF SUMS OR AVGS OF RANDOM VARIABLES.]

BINOMIAL SETUP: n INDEP ATTEMPTS.

eg SHIPMENT OF 10,000 PARTS

SAMPLE $n = 60$

SUPPOSE 20% OF PARTS ARE FAULTY

SAMPLE PARTS $\approx$ INDEPENDENT

"WHAT IF" X = # FAULTY IN SAMPLE

$$\text{BIN } P(X = 10) = P(\underbrace{FF\ F}_{10}GG \cdot G) = .2^{10} .8^{50}$$

$$+ P(\underbrace{F \cdot\cdot F}_{9}GFG \cdot G) = .2^{10} .8^{50} \text{ etc}$$

$$= \frac{60!}{10!\,50!} (.2^{10}\ .8^{50})$$

IF I WANT YOU TO USE THIS IT WILL BE DISPLAYED ON EXAM

GEN'L ((BIN)) $\frac{n!}{x!(n-x)!} p^x q^{n-x}$, $q = 1-p$

$x = 0, 1, \ldots, n$

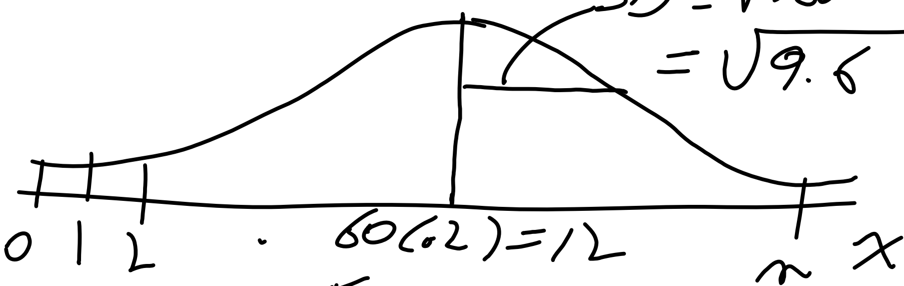
BUT NICE PART

"WHAT IF"

$\approx$ DIST OF X

$SD = \sqrt{Var}$

$= \sqrt{9.6} \approx 3^+$



$EX = np$

$P(X \text{ IN } 12 \pm 3) \underset{\text{NAIVE}}{\approx} .68$

12 ± 6 .95

2 SD FROM MEAN

$VarX = npq = 60 \cdot .2 \cdot .8$

$= 9.6$

FINDING $X = 21$ VERY FAR OUT! ((IDEA: TESTING))

POISSON (CAN THINK OF IT AS BINOMIAL $n \sim \infty$, $p \sim 0$)

(SAY) OBSERVE (≈) 16 CANCELLATIONS.

$EX = \lambda = 16$ (PAST EXPERIENCE)

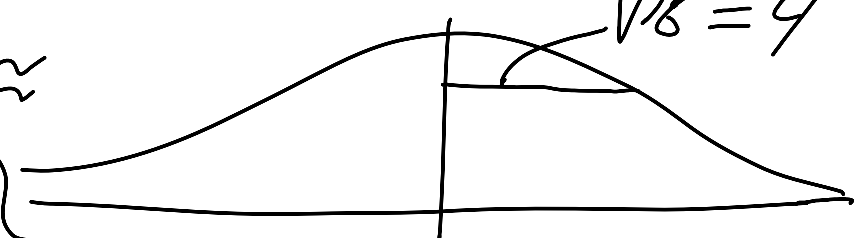
IF POISSON $P(X = 14) = p(14) = e^{-16}\, 16^{14}/14!$

GEN'L $p(x)_{\text{POISSON}} = e^{-\lambda} \lambda^x / x!$ $\quad \lambda > 0$, $x = 0, 1, \ldots$ ad inf

ALSO FOR POISSON $\text{Var} = \lambda$ (= MEAN)

(RECALL MEAN BINOMIAL $np = \lambda_{\text{POISSON}}$, Var BINOMIAL $\underbrace{np}_{\lambda}\underbrace{q}_{1}$ if $p \sim 0$)

ABOVE EXAMPLE SEE THRU NORMAL APPROX ≈



SO $P(X \text{ IN } 16 \pm 4) \overset{\text{NAIVE}}{\approx} .68$

$P(X \text{ IN } 16 \pm 8) \approx .95$

SETUP OF SUMS OF INDEP r.v.

PLAY $EX = 7.4$ $\sigma_X = SD\,X = 11.3$

PAST $\bar{x} \sim 7.4$ $s = 11.3$ EXPERIENCE

MANY INDEPENDENT PLAYS — $X_1, \ldots, X_{40}$

$T = X_1 + \cdots + X_{40}$

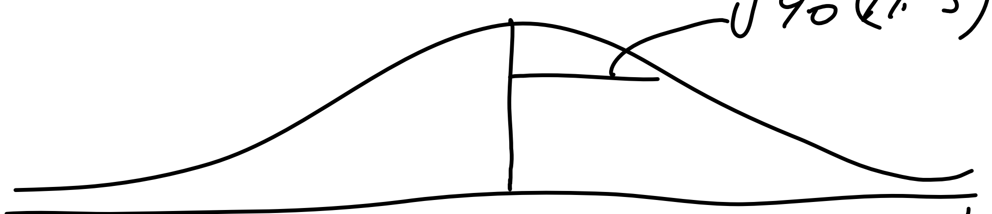
$ET = 40(7.4)$

$\text{Var}\,T = 40\,\text{Var}\,X_1 = 40\,(11.3)^2$

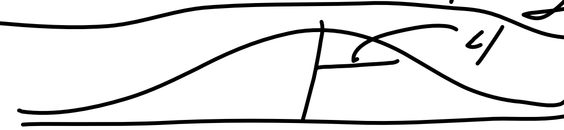
$\sigma_T = \sqrt{40(11.3)^2} = \sqrt{40}\;11.3 \approx 72$

CLT

$\approx$ DIST T



ANOTHER: AVG 16 HITS (POISSON)



ANOTHER (TOTAL OF win.)

AVG 18 PER PLAY $EX = 18$

$\text{Var} X = 25$ ($\text{SD} X = 5$)

$T = X_1 + \cdots + X_{100}$ INDEP PLAYS

DIST $T \approx$

$ET = 18(100) = 1800$

$P(X \text{ IN } 1800 \pm 50) \approx .68$

$1800 \pm 100 \approx .95$

PROBABILITY

TREE: $\{6R\ 3G\ 2B\}$ $\{11R\ 4G\ 1B\}$

I $\qquad$ II

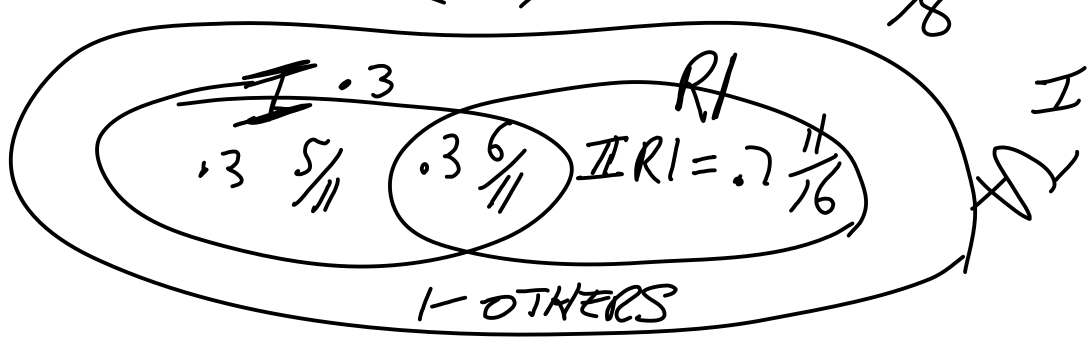
PICK BOX $P(\mathrm{I}) = .3$ $\quad P(\mathrm{II}) = .7$

$$P(R1) \overset{\text{TOTAL}}{=} P(\mathrm{I}\, R1) \underset{\text{OR}}{+} P(\mathrm{II}\, R1)$$

$$\underset{\text{MULT}}{=} P(\mathrm{I})\, P(R1 \mid \mathrm{I}) \underset{\text{OR}}{+} P(\mathrm{II})\, P(R1 \mid \mathrm{II})$$

IF (GIVEN)

$$= .3\left(6/11\right) + .7\ \frac{11}{16}$$



$\mathrm{I}^c = \mathrm{II}$