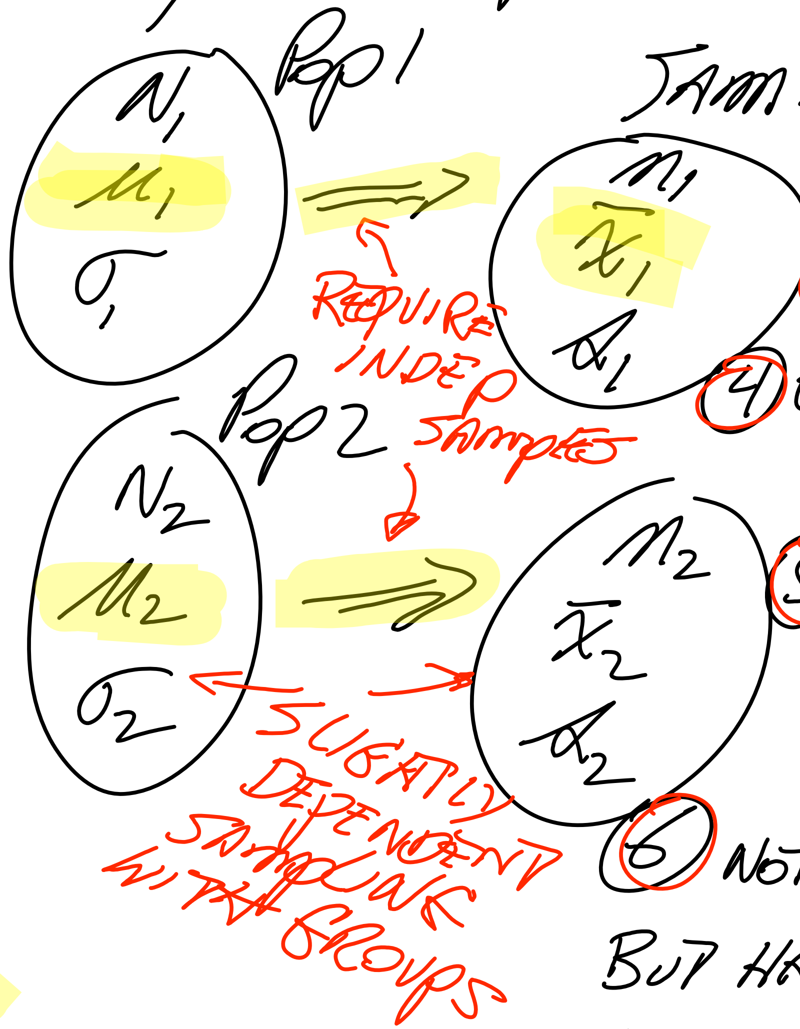
STT200 1-21-09a

TODAY: C.I. for $\mu_1 - \mu_2$ (Difference of pop means).



JUST FOR POP 1 SAMPLE

1. EST'D μ_1 IS $\bar{x}_1$
2. EST'D σ_1 IS s_1
3. EST'D STD ERROR $\bar{x}_1$ IS $\frac{s_1}{\sqrt{n_1}}\sqrt{\frac{N_1-n_1}{N_1-1}}$
4. EST'D MARGIN OF ERROR OF $\bar{x}_1$ IS $1.96\,\frac{s_1}{\sqrt{n_1}}\sqrt{\frac{N_1-n_1}{N_1-1}}$
5. 95% CI for μ_1 IS

$$\bar{x}_1 \pm 1.96\,\frac{s_1}{\sqrt{n_1}}\sqrt{\frac{N_1-n_1}{N_1-1}}$$

6. NOTE: CI ALWAYS CONTAINS $\bar{x}_1$!! BUT HAS ~95% CHANCE OF CONTAINING μ_1

EXAMPLE: $n = 3$ DATA $\{2, 6, 10\}$

SUPPOSE $N = 400$ (POP SIZE)

$$\bar{x} = \frac{2+6+10}{3} = \frac{18}{3} = 6$$

$$s = \sqrt{\frac{(2-6)^2+(6-6)^2+(10-6)^2}{3-1}}$$ SAMPLE S.D.

$$= \sqrt{\frac{16+0+16}{2}} = \sqrt{16} = 4$$

n NOT LARGE $\Rightarrow$ "95% CI" $\bar{x} \pm 1.96 \frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$ IS NOT KNOWN TO APPLY — BUT DOING IT FORMALLY

$$6 \pm \cancel{1.96}^{\,4.303} \frac{4}{\sqrt{3}} \cancel{\sqrt{\frac{400-3}{400-1}}} \approx 1$$

3-1	4.303
∞	1.96
	95%

CANNOT MAKE

1

$n << N$

NOTE (SEE 1-23-09) IF POPULATION $\approx$ NORMAL CAN THEN QUOTE EXACT 95% CI

WHAT TO DO ABOUT ESTIMATING $\mu_1 - \mu_2$:

MIGHT THINK

$$\bar{x}_1 \pm \underbrace{\text{EST}^{D}\ \text{MOE}_1}_{\text{E MOE}_1}$$

$$1.96\,\frac{\sigma_1}{\sqrt{n_1}}\sqrt{\frac{N_1-n_1}{N_1-1}}$$

(or 4.303 etc)

$$\bar{x}_2 \pm \underbrace{\text{EST}^{D}\ \text{MOE}_2}_{\text{E MOE}_2}$$

MELD TO OBTAIN EFFICIENT USE OF INFO.

VOILA! EST^{D} MOE FOR $\bar{x}_1 - \bar{x}_2$

INDEPENDENT

$$\text{E MOE OF } \bar{x}_1 - \bar{x}_2 \pm \sqrt{\text{E MOE}_1^{\,2} + \text{E MOE}_2^{\,2}}$$

PYTHAGORAS! WOW!!

EXAMPLE.

TWO STUDIES

① $\bar{x}_1 = 2.34 \quad s_1 = 1.42$

$n_1 = 40 \quad N_1 \sim \infty$

② $\bar{x}_2 = 2.62 \quad s_2 = 1.31$

$n_2 = 50 \quad N_1 \sim \infty$

95% CI for $\mu_1 = 2.34 \pm 1.96 \dfrac{1.42}{\sqrt{40}} \, 1$

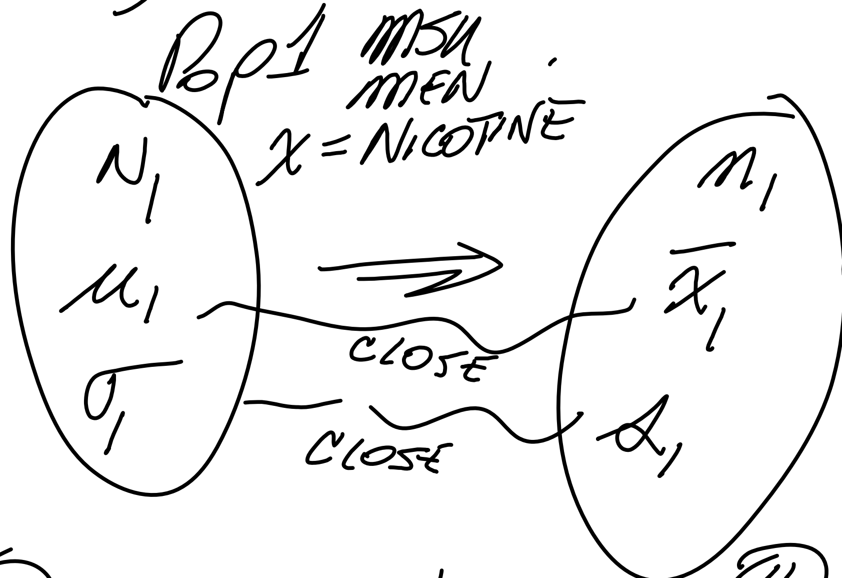
95% CI for $\mu_2 = 2.62 \pm \underbrace{1.96 \; 1.31/\sqrt{50}}_{\text{EMOE}}$

95% CI for $\mu_1 - \mu_2$

$$(2.34 - 2.62) \pm \sqrt{\underbrace{\text{EMOE}_1^2} + \underbrace{\text{EMOE}_2^2}}$$

STP200 1-21-09b

TODAY: CI for $\mu_1 - \mu_2$ (INDEP SAMPLES)



① $\bar{x}_1$ POINT EST OF μ_1

② s_1 POINT EST OF σ_1

③ ESTIMATED STANDARD ERROR OF $\bar{x}_1$ IS $\dfrac{s_1}{\sqrt{n_1}}\sqrt{\dfrac{N_1 - n_1}{N_1 - 1}}$

④ ESTIMATED MARGIN OF ERROR OF $\bar{x}_1$ IS

$$1.96\,\frac{s_1}{\sqrt{n_1}}\sqrt{\frac{N_1 - n_1}{N_1 - 1}}$$

⑤ 95% CI for μ_1

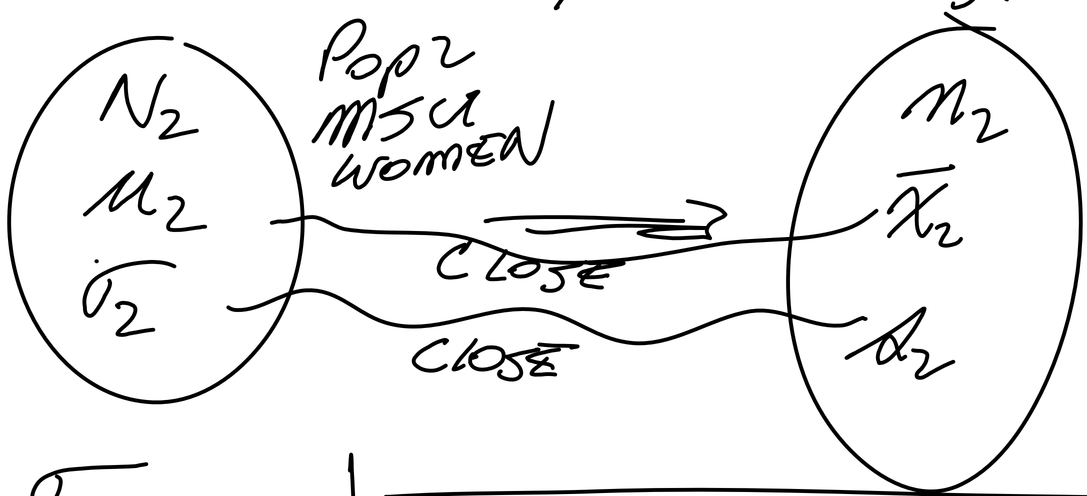
$\bar{x}_1 \pm EMOE_1$

$$\bar{x}_1 \pm 1.96\,\frac{s_1}{\sqrt{n_1}}\sqrt{\frac{N_1 - n_1}{N_1 - 1}}$$

CLAIM: ~95% OF 95% CI COVER THEIR TARGET.

LIKEWISE

x = NICOTINE



95% CI for μ_2

$$\bar{x}_2 \pm 1.96 \frac{\sigma_2}{\sqrt{n_2}} \sqrt{\frac{N_2 - n_2}{N_2 - 1}}$$

EST MOE for $\bar{x}_2$

= EMOE$_2$

σ

GREEK "SIGMA"

l.c.

Σ

u.c. SIGMA.

$$\sigma = \sqrt{\sum_{1}^{N} (x_i - \mu)^2 / N}$$

EXAMPLE: $n = 3$ (TOO SMALL!)

DATA $\{2, 6, 10\}$.

$$\bar{x} = \frac{2+6+10}{3} = 18/3 = 6$$

$$s = \sqrt{\frac{(2-6)^2 + (6-6)^2 + (10-6)^2}{3-1}} = \sqrt{\frac{32}{2}} = 4$$

"95% CI for μ": $\bar{x} \pm 1.96 \frac{4}{\sqrt{3}} \sqrt{\frac{N-3}{N-1}} \approx 1$

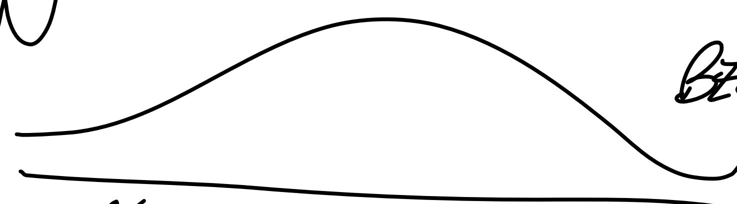
$n << N$

NOTE: $n = 3$ SO SMALL WE CANNOT GUARANTEE
CHANCE 95% CI COVERING μ $\approx .95$.

<u>HOWEVER</u> CHECK LEC. OUTLINE 1-23-09

FIND ~~THAT~~ CAN USE $\bar{x} \pm \underset{t\text{-score}}{\cancel{1.96}}\ \frac{s}{\sqrt{n}}$ 1

PROVIDED THE POPULATION SCORES $X_1 \ldots X_N$
HAVE DISTRIBUTION



BELL CURVE

PER DATA ABOVE, IF POP^N IS NORMAL

THEN

$\bar{x} = 6$ $n = 3$

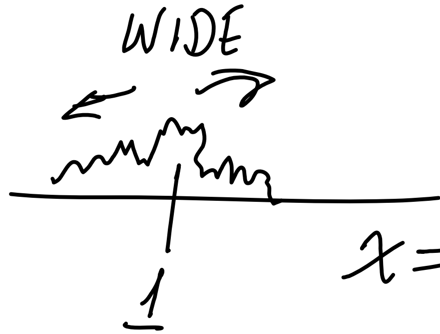
$s = 4$

CHANCE $\bar{x} \pm (t\text{-score})\frac{s}{\sqrt{n}}$ COVERS μ EXACT $= .95$

$6 \pm 4.303\ 4/\sqrt{3}$

DF	
$3-1=2$	4.303
∞	1.96

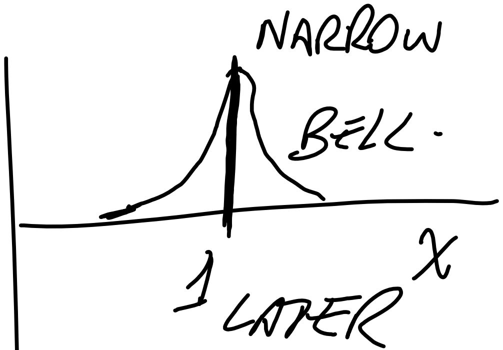
NOTE:



x = ROUNDNESS

EARLY PROCESS

UNINTENDED SOURCES OF VARIATION



CAN USE SAMPLES AS SMALL AS $n=2$

TODAY RECALL - ISSUE OF CI for $\mu_1 - \mu_2$

HAVE, LET'S SUPPOSE,

95% CI for μ_1: $1.63 \pm .72$ @ $\bar{x}_1$ & MOE_1 ~1.96 IN HERE

" " " μ_2: $1.38 \pm .61$ @ $\bar{x}_2$ $n_1 \sim \infty$

THEN

95% CI for $\mu_1 - \mu_2$: $1.63 - 1.38 \pm \sqrt{.72^2 + .61^2}$