STT 200 5:30pm 1-13-10

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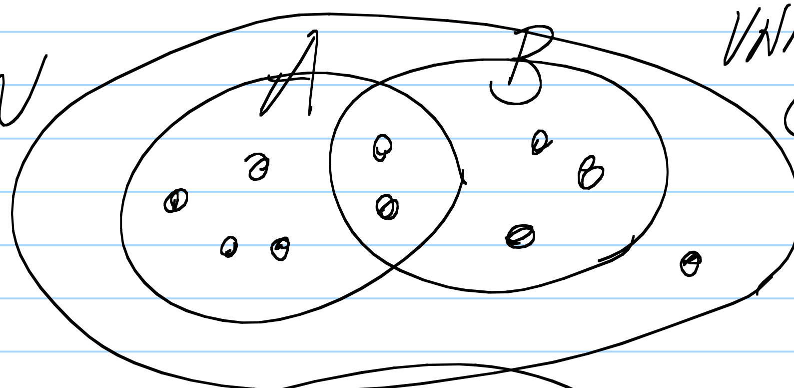
Note Title 1/13/2010

TODAY CH 15. RECALL CH 14

CLASSICAL PR MODEL.

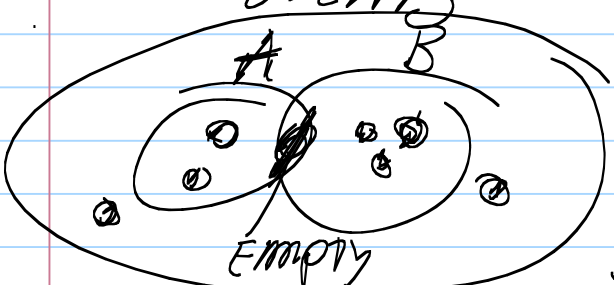
VENN

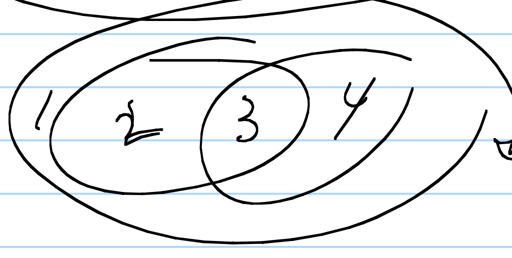


UNIVERSE (SAMPLE SPACE)

ALL 10 EQ PR.

DISJOINT EVENTS





$P(A \underset{or}{\cup} B) = P(A) + P(B) - P(A \cap B)$

2 3 4   2 3   3 4

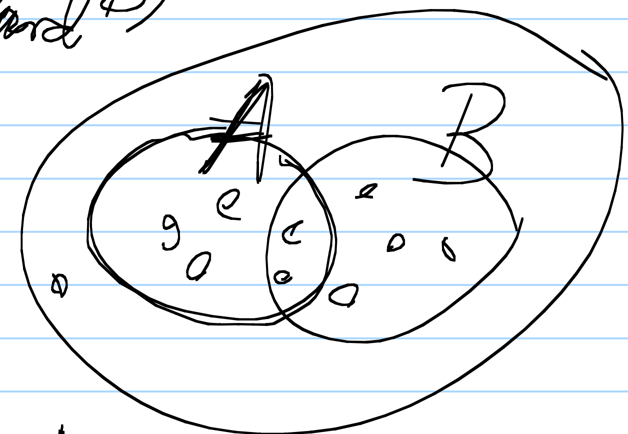
DISJOINT EVENTS: $P(A \underset{or}{\cup} B) = P(A) + P(B) = 2/7 + 3/7$

CH 15 GEN'L ADD'N RULE

$$P(A \underset{\text{or}}{\cup} B) = P(A) + P(B) - P(A \underset{\text{and}}{\cap} B)$$

ATTACK $P(A \underset{\text{and}}{\cap} B)$

CLASSICAL $\dfrac{\# A \cap B}{\# \text{TOTAL}} = \dfrac{\# A}{\# \text{TOTAL}} \boxed{\dfrac{\# A \cap B}{\# A}}$



$$P(A \underset{\text{and}}{\cap} B) = P(A) \boxed{\frac{2}{5}} \underset{\text{DEF}}{=} P(B \underset{\text{IF}}{|} A)$$

$$\frac{\# A \cap B}{\# A} = \frac{\# A \cap B / \# \text{TOTAL}}{\# A / \# \text{TOTAL}} = \frac{P(A \cap B)}{P(A)}$$

CONDITIONING EVENT

SO INDEED COND'L PROB'Y FOR B GIVEN THAT A WAS OCCURRED

$$P(B \underset{IF}{|} A) \overset{DEF}{=} \frac{P(A \cap B)}{P(A)}$$

IF THIS IS JUST $P(B)$

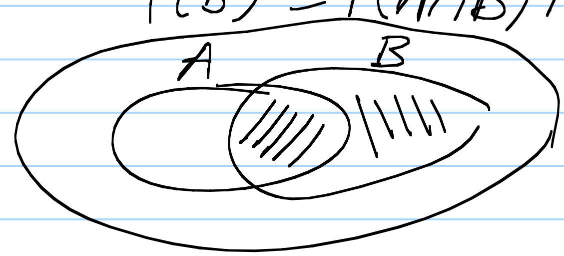
IT WOULD THEN BE EQUIV TO

$$P(A \cap B) = P(A)\,P(B)$$

CH 14. MULT RULE IF EVENTS ARE INDEP.

ILLUSTRATION OF RULES ADD^N, MULT, TOTAL PROB^Y

$$P(B) = P(A \cap B) + P(A^c \cap B)$$



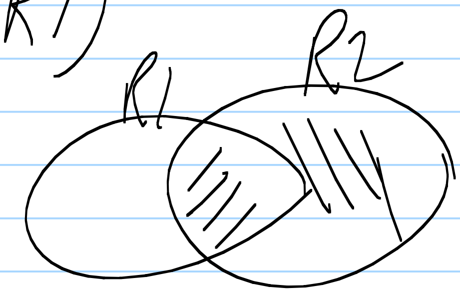
BALLS 6 R 3 G 4 Y

$$P(R1) = 6/13.$$

$$P(R2 \,|_{\text{IF}}\, R1) = 5/12$$

$$P(R1 \cap_{\text{and}} R2) = P(R1)\,P(R2 \,|_{\text{IF}}\, R1)$$

$$= 6/13 \;\; 5/12$$



$$P(R1^{c} \cap_{\text{and}} R2) = 7/13 \;\; 6/12$$

ADD THESE $P(R2) = \frac{6}{13}\,\frac{5}{12} + \frac{7}{13}\,\frac{6}{12} = \frac{\overset{=6\times 12}{72}}{13\cdot 12} = \frac{6}{13}$

SAME AS $P(R1)$

REDO ABOVE BUT DRAW WITH REPL.

INTUITION: $P(R1) = 6/13$, $P(R2) = 6/13$.

$P(R1 \cap_{\text{and}} R2) = P(R1)\, P(R2 \mid_{\text{IF}} R1) = \frac{6}{13} \cdot \frac{6}{13}$ (INDEP)

$P(R1^c \cap_{\text{and}} R2) = P(R1^c)\, P(R2 \mid \cancel{R1^c}) = \frac{7}{13} \cdot \frac{6}{13}$

SAME AS $P(R1)$ $\quad P(R2) = \text{TOTAL} = 6/13$

INDEPENDENCE OF A, B

DEF: $P(A \cap_{\text{IND}} B) \stackrel{\text{IND}}{=} P(A)\, P(B)$

EXCEPT FOR CASE $P(A) = 0$ THIS SAYS

$P(B \mid_{\text{IF}} \cancel{A}) = P(B)$ CROSS OUT IF INDEP.

IN TABLES, INDEP IS SEEN AS PROPORTIONALITY

	AA	Aa	aa
M	10	20	30
F	5	10	15

$$P(AA \,|_{IF}\, M) = \frac{P(AA\, M)}{P(M)} = \frac{10}{60}$$

$$P(AA \,|_{IF}\, F) = \frac{5}{30}$$ SAME

PROPORTIONALITY IS THE LOOK OF INDEPENDENCE!

TREE DIAGRAM

TOLD $P(OIL) = .2$

POS FOR OIL

$P(+ \,|_{IF}\, OIL) \sim .9$ NICELY LARGE

$P(+ \,|_{IF}\, OIL^c) \sim .3$ NICELY SMALL

$= .2$

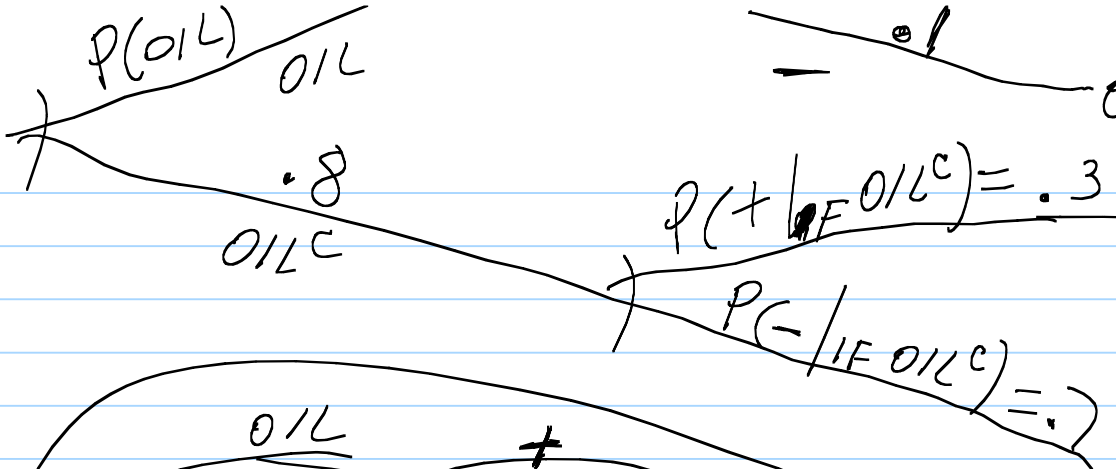
$P(+ \,|_{IF}\, OIL) = .9$

$+$

$P(- / OIL) =$

and

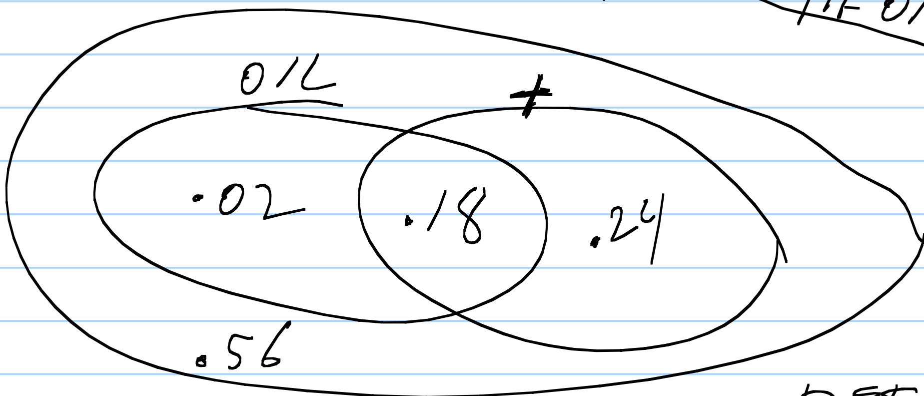
$OIL+ \quad .2 \cdot .9 = .18 \;\}\; .2$



OIL + $.2 \cdot .1 = .02$ } P(OIL)

OIL^C and + $.8 \cdot .3 = .24$

OIL^C − $.8 \cdot .7 = .56$

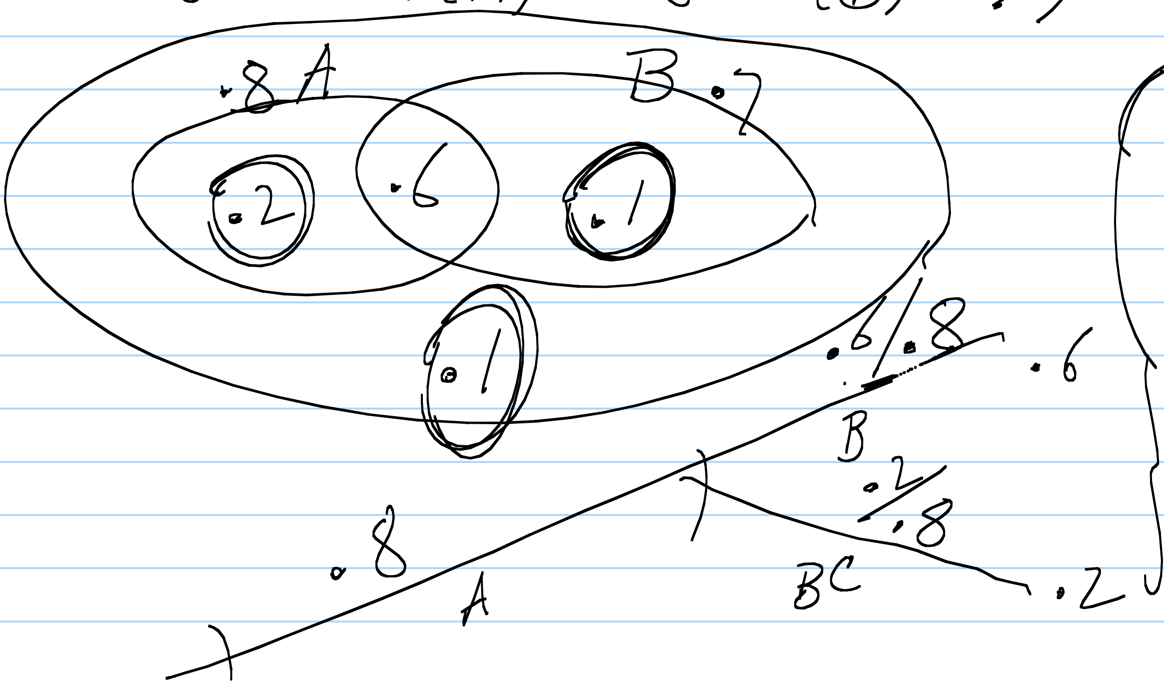


BAYES $P(OIL \mid_{IF} +) \overset{DEF}{=} \dfrac{P(OIL \text{ and } +)}{P(+)} = \dfrac{.18}{.18 + .24}$

COMPARE THIS TO INITIAL $P(OIL) = .2$

$$\therefore P(\text{OIL} \underset{\text{IF}}{|} -) = \frac{P(\text{OIL} \overset{\text{and}}{} -)}{P(-)} = \frac{.02}{.02 + .56}$$

#2. GIVEN $P(A) = .8$ $P(B) = .7$ $P(A \underset{\text{and}}{\cap} B) = .6$ ✓



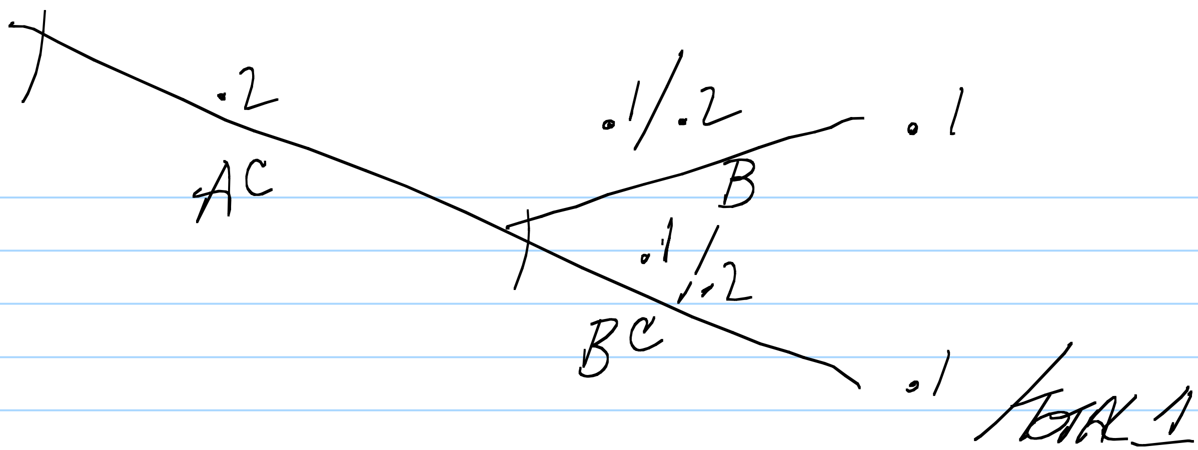
$P(A) = .8$

(IF A, B WERE INDEP.

$P(A \underset{\text{and}}{\cap} B) = P(A)P(B)$

$= .56$

NOT)


.2
AC
.1/.2
.1
B
.1/.2
BC
.1
Total 1