STT 200 3pm 1-20-10

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Note Title 1/20/2010

LEAD OFF W/ OVERVIEW CH 6

DEF OF RANDOM VARIABLE:

A NUMERICAL FUNCTION ON THE OUTCOMES OF A PROBABILITY EXPERIMENT.

eg JACK + JILL \$1 \$1 \$5 JACK DRAWS FIRST.

RANDOM VARIABLE

J = "AMOUNT DRAWN BY JACK"

outcomes	\$1	\$1	\$5
	a	b	c
value of J	1	1	5

NOTION OF RANDOM VARIABLE

NOTATION X, Y etc. x, y

$EX = \sum_{\text{STUDENTS}} WT / \#\text{STUDENTS}$

SAME = ADJ. $\sum_{v} v \, p(v)$

DISTINCT POSS WT (NEAREST POUND)

VARIANCE

$\sum (v - EX)^2 p(v)$

(DEVIATIONS FROM AVG)^2

FIND $P(J=1) = \frac{\# \text{ FAVORABLE}}{\# \text{ TOTAL}} = 2/3 \quad P(J=5) = \frac{1}{3}$

$$\left[\begin{array}{lcc} \text{VALUES } j: & 1 & 5 \\ p(j): & 2/3 & \frac{1}{3} \end{array}\right]$$

DISTRIBUTION OF RANDOM VARIABLE

NOTION OF "PROBABILITY WEIGHTED AVERAGE VALUE"

$$EJ = \sum_j \underbrace{j}_{\text{POSSIBLE VALUE OF } J} p(j) = 1\left(\frac{2}{3}\right) + 5\left(\frac{1}{3}\right) = 7/3$$

SAY $EJ = 7/3$

SAY "EXPECTED VALUE OF J IS 7/3"

BUT JACK EITHER GETS \$1 OR \$5.

RELEVANCE OF EJ IS THAT $\frac{J_1 + J_2 + \cdots + J_{10000}}{10000} \sim 7/3$

IMPORTANT: IS A SENSE TO BE DESCRIBED,

RANDOM SUM $V1 + \cdots + V10000$

RETURN OF VENTURE 1 ... VENTURE 10000

TRACK CLOSELY WITH $EV1 + \cdots + EV10000$

SOME CONDITIONS REQUIRED

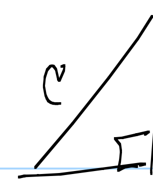
#4. ON RECITATION 1-26-10

LOTTERY				
VALUES x	20	-5	0	
$p(x)$	0.2	0.4	0.4	(NOT 0.6 AS ORIG POSTED)

4a. CALC. $EX \overset{DEF}{=} \sum_x x\,p(x) = 20(.2) - 5(.4) + 0(.4)$

VALUE $= 4 - 2 + 0 = 2$

4b. $\sum_x (x - EX)^2 p(x)$ RECALL $EX = 2$



$= (20-2)^2 (0.2) + (-5-2)^2 (0.4) + (0-2)^2 (0.4) = 86$ $a^2+b^2=c^2$

x EX $p(x)$

TERMINOLOGY: Variance of X $\stackrel{DEF}{=} \sum_x (x-EX)^2 p(x)$

CAN BE WRITTEN $= E(X-EX)^2$

IMPORTANT PROPERTY Variance of X $\stackrel{ALSO}{=} E(X^2) - (EX)^2$

$EX = 2$ RECALL so $(EX)^2 = 4$

$E(X^2) = \sum_x x^2 p(x) = 20^2 (.2) + (-5)^2 .4 + 0^2 (.4) = 90$

SO TRULY $\underbrace{\text{Variance of } X}_{\text{BY THE ALTERNATE METHOD "AVG OF SQUARES" — SQUARE OF PR WTD AVG.}} = 90 - 4 = 86$

90 IS AVG OF SQUARES $E(X^2) = \sum_x x^2 p(x)$

4 IS $(EX)^2 = \left(\sum_x x\, p(x)\right)^2$

4c. STANDARD DEVIATION $\stackrel{DEF}{=} \sqrt{\text{Variance}}$

$= \sqrt{86} = 9.27362$

#5. LOTTERY HAS RANDOM RETURN X, WITH

$EX = -\$0.10$ STANDARD DEVIATION $X = \$0.80$

WILL PLAY 100 TIMES INDEPENDENTLY

5a. **CLAIM:** $E(\text{TOTAL OF 100 PLAYS}) = E(X_1 + X_2 + \cdots + X_{100})$

$= EX_1 + \cdots + EX_{100} = 100(-\$0.10) = \underline{-\$10.00}$

5b. $\text{Variance}(X_1 + \cdots + X_{100}) = \text{Var}\,X_1 + \cdots + \text{Var}\,X_{100}$

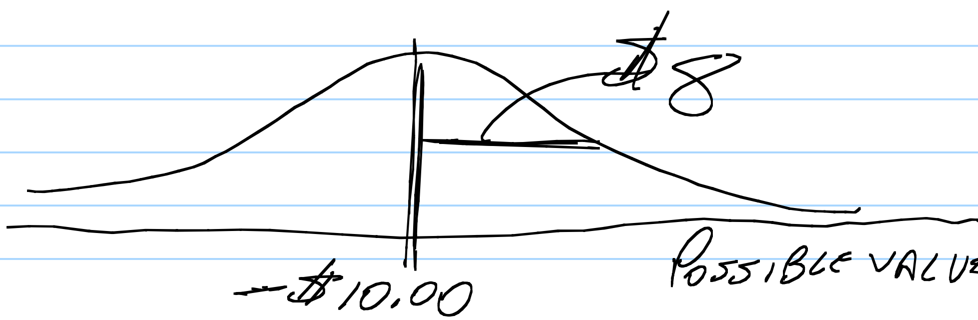
FOR INDEPENDENT PLAYS

$= 100(\$.80)^2 = 64$

SO STANDARD DEVIATION OF $(X_1 + \cdots + X_{100}) = \sqrt{64} = \underline{8}$

5c. SKETCH THE APPROXIMATE APPEARANCE OF THE TOTAL FOR 100 INDEPENDENT PLAYS.

$\approx$

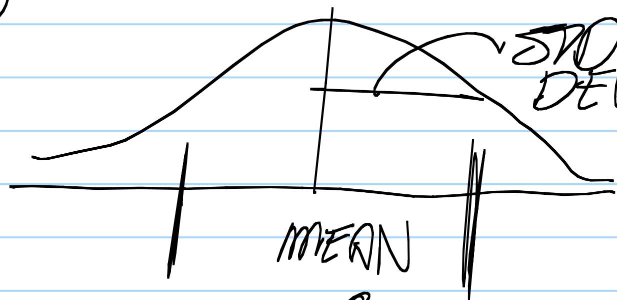


-$2.00

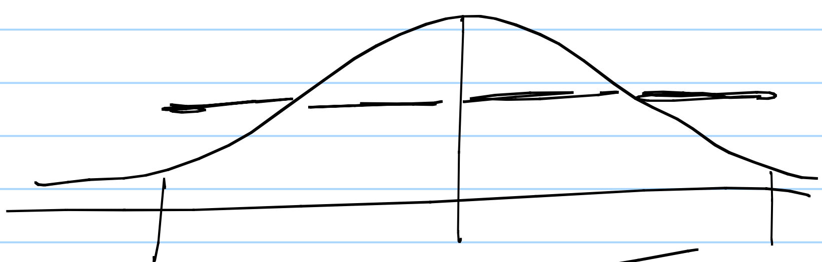
- $10.00 - 8 $-10.00 + 8 HAS ≈ 68%

-$18.00

PROBABILITY

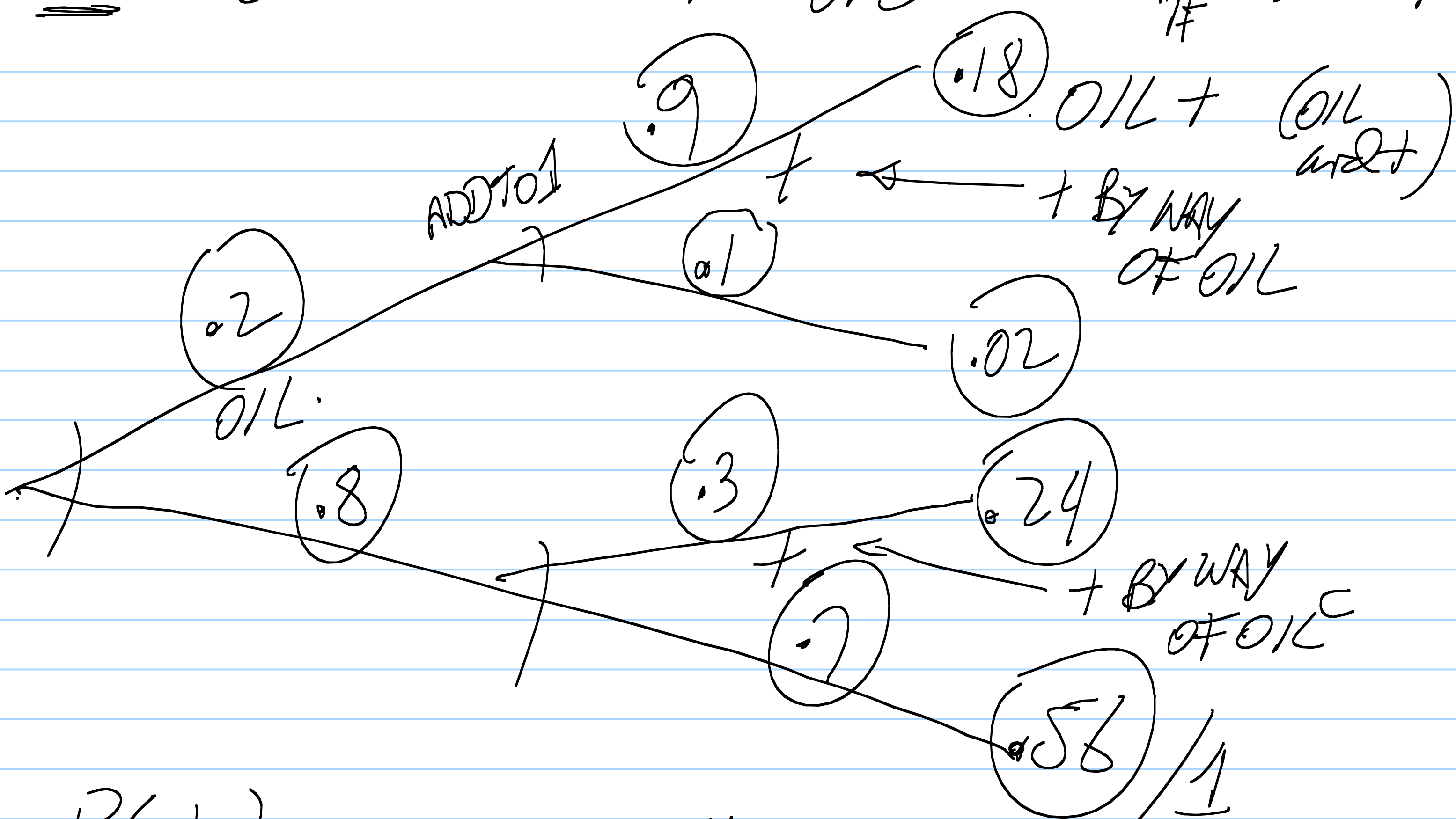


68%



95%

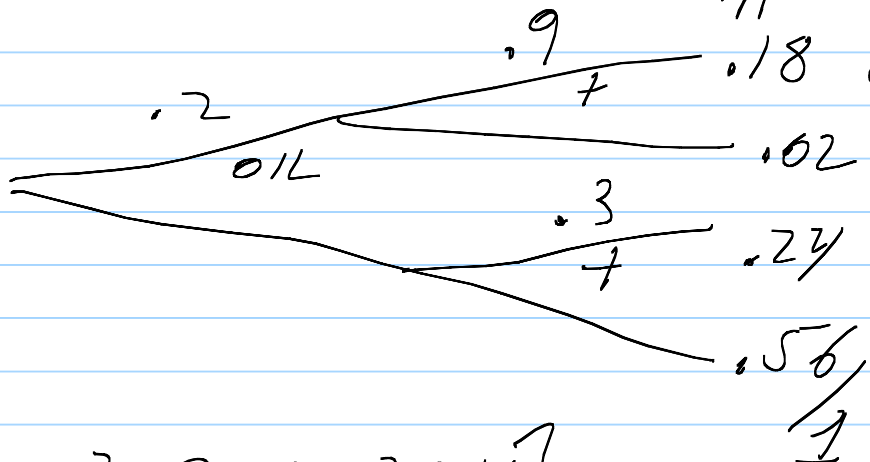
#3. OIL $P(OIL) = 0.2$ $P(+ \mid_{IF} OIL) = 0.9$ $P(+ \mid_{IF} OIL^c) = 0.3$



$P(+) = .18 + .24$

BAYES – UPDATE $P(OIL \mid_{IF} +) \underset{DEF}{=} \frac{P(OIL+)}{P(+)} = \frac{.18}{.18+.24}$

3b



$-80-200+900$

$-80-0+0$

$-80-200+0$

$-80-0+0$

$\sum x\,p(x) = -\$2$

COSTS 200 TO DRILL
COSTS 80 TO TEST
REWARD 900

$E(\text{NET } I) = (-200+900)(.2) + (-200+0)(.8) = -20$

POLICY I "JUST DRILL"

POLICY $\underline{II}$ "TEST BUT ONLY DRILL IF TEST IS +"