STT 200 5:30pm 1-20-10

Note Title 1/20/2010

INTRODUCE RANDOM VARIABLE

CH 16

OVERVIEW OF SOME ESSENTIALS

- EXPECTATION OF RANDOM VARIABLE
- Variance + STANDARD DEVIATION OF RANDOM VARIABLE
- BELL CURVE
- ROLE OF ABOVE IN DESCRIBING BEHAVIOR OF SUMS OF INDEPENDENT RANDOM VARIABLES.

DO LAST PARTS OF RECITATION 1-26-10.

DEFINITION: RANDOM VARIABLE IS NUMERICAL FUNCTION ON THE OUTCOMES OF A PROBABILITY EXPERIMENT.

eg JACK & JILL $1 $1 $5

a b c

RANDOM VARIABLE J = "AMT JACK GETS"

POSSIBLE VALUES OF J

j	1	5
$p(j)$	$2/3$	$\frac{1}{3}$

PROBABILITY DISTRIBUTION OF RANDOM VARIABLE J

SPEAK OF $P(J=1) = 2/3$ $P(J>2) = P(J=5) = \frac{1}{3}$

SPEAK OF $\underbrace{EJ}_{\text{"EXPECTATION OF RANDOM}} = \sum_j j\, p(j) = 1\left(\frac{2}{3}\right) + 5\left(\frac{1}{3}\right) = 7/3$

LAW AVERAGE

$1\left(\frac{2}{3}\right)+5\left(\frac{1}{3}\right) \approx (\#1\text{'s} + 5(\#5\text{'s}))/10000$

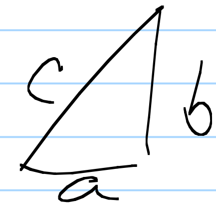
DRAW 10000 TIMES. GET SAMPLE AVG.

VARIANCE OF RANDOM VARIABLE $\stackrel{DEF}{=} \sum_j \left(j - \frac{7}{3}\right)^2 p(j)$

$$= \left(1 - \frac{7}{3}\right)^2 \left(\frac{2}{3}\right) + \left(5 - \frac{7}{3}\right)^2 \left(\frac{1}{3}\right)$$

GEN'L WRITE $E(J - EJ)^2$

PROBABILITY WEIGHTED AVERAGE



$a^2 + b^2 = c^2$

STANDARD DEVIATION OF RANDOM VARIABLE:

STD DEV OF J $\stackrel{DEF}{=} \sqrt{\text{VARIANCE OF J}}$

ALTERNATIVE CALCULATION OF VARIANCE:

$$E(J - EJ)^2 = E(J^2) - (EJ)^2$$

SO Variance of J BY THIS OTHER WAY

$$= E(J^2) - (E J)^2$$

$$= 9 - (7/3)^2 = \frac{32}{9}$$

$$E(J^2) = 1^2\left(\frac{2}{3}\right) + 5^2\left(\frac{1}{3}\right) = \frac{27}{3} = 9$$

SO STANDARD DEVIATION OF J IS $\sqrt{32/9}$

#5. SHOWS RELEVANCE OF $E X$ AND STD DEV OF X TO AGGREGATE RETURN OF A LARGE NUMBER OF INDEPENDENT PLAYS OF THE LOTTERY.

eg

x	20	-5	0
$p(x)$	.2	.4	.4 (NOT 0.6)

#4

$$EX = 20(.2) - 5(.4) + 0(.4) = 4 - 2 = 2$$

$$E(X_1 + \cdots + X_{100}) \overset{\text{ALWAYS}}{=} E(X_1) + \cdots + E(X_{100})$$

X_1: RETURN ON PLAY 1

X_{100}: RETURN ON PLAY 100

$$= 2 + \cdots + 2 = 200$$

THIS RULE DOES NOT REQUIRE INDEPENDENCE.

WHAT ABOUT VARIANCE OF $X_1 + \cdots + X_{100}$

ANS SINCE THEY ARE INDEPENDENT PLAYS

$$\text{Var}(\text{SUM}) = \text{SUM OF VARIANCES} = 100(\text{Var}\,X_1)$$

REQUIRES INDEPENDENCE.

Variance of X_1: $EX^2 - (EX)^2$ $\quad EX = 2$

$$E(X^2) = 20^2(.2) + (-5)^2 .4 + 0^2(.4) = 80 + 10 = 90$$

SO $\underset{\text{ONE PLAY VARIANCE}}{\text{Var } X} = E(X^2) - (EX)^2 = 90 - 2^2 = 86.$

SO STANDARD DEVIATION OF X IS $\sqrt{86} = 9.27362$

Sum up: LOTTERY (ONE PLAY)

x	20	-5	0
$p(x)$	.2	.2	.4

$\Rightarrow$ $\underset{\text{SINGLE PLAY}}{\underline{EX = 2}}$, Var $X = 86$, STANDARD DEVIATION $= \sqrt{86} = 9.27$

$\Rightarrow$ (RULES) $E(\text{TOTAL OF 100 PLAYS}) = 100(2) = 200$

$\text{Var}(\text{TOTAL OF 100 INDEP PLAYS}) = 100(86)$

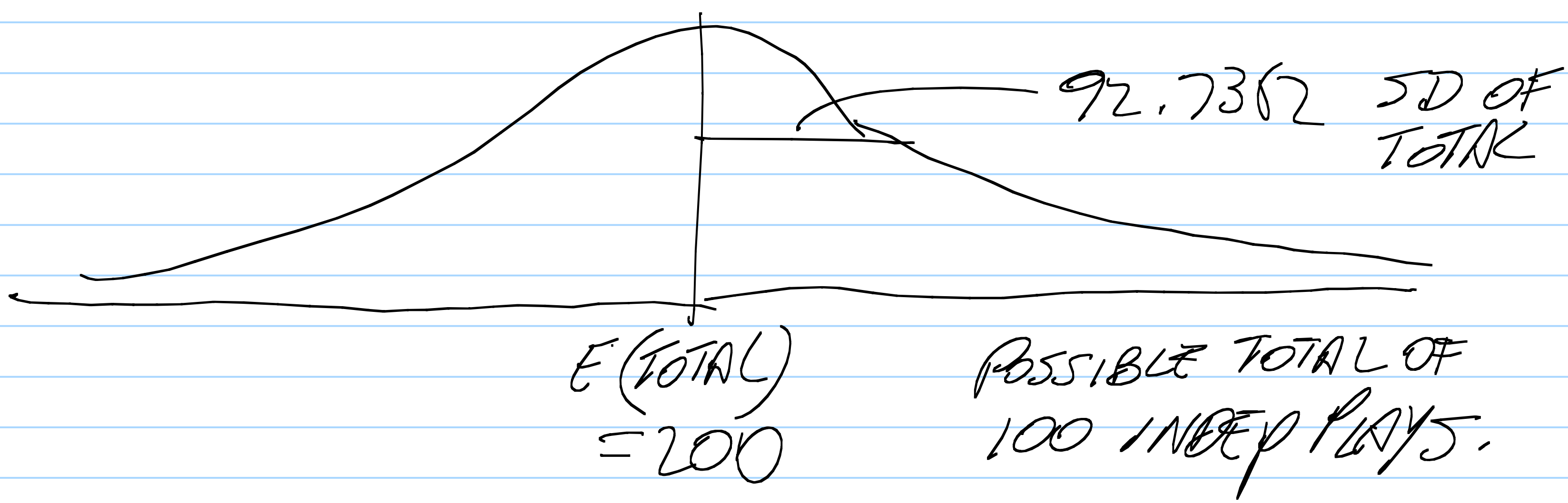
$$SD(\text{TOTAL OF 100 INDEP PLAYS}) = \sqrt{100 \cdot 86}$$

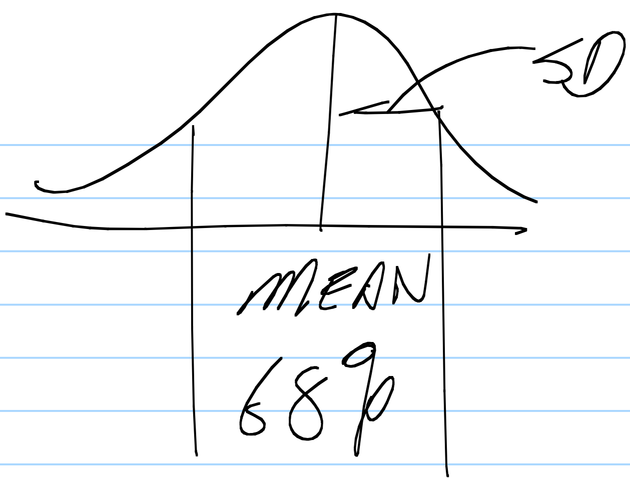
STANDARD DEVIATION

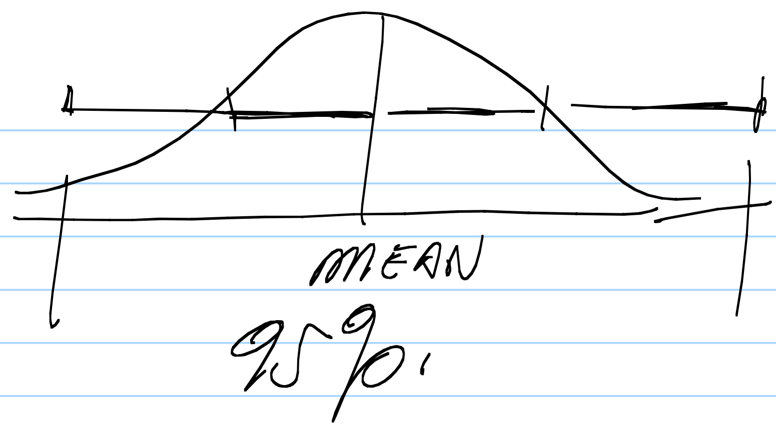
$$= 10(9.27362)$$
$$= 92.7362$$

NOW APPLY A BELL CURVE APPROX.

$\approx$







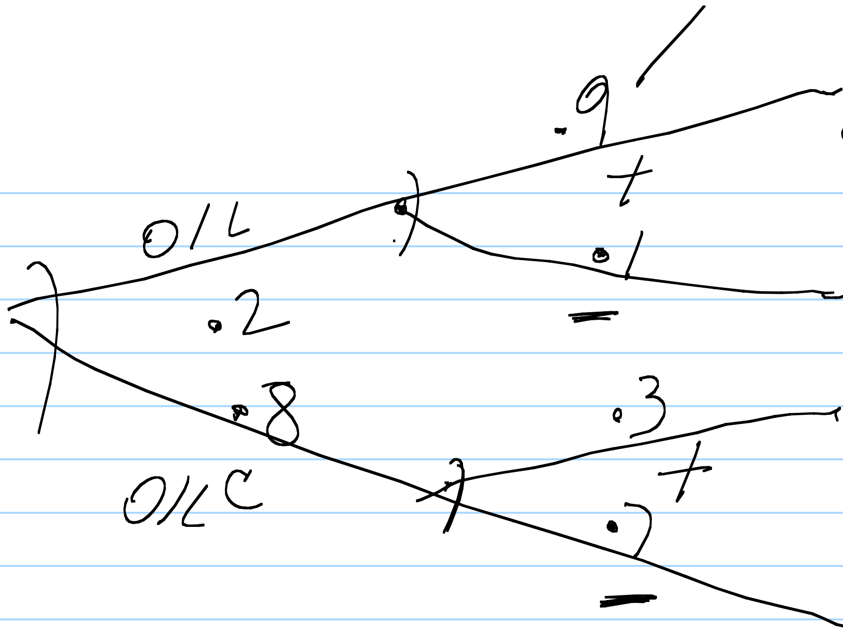
SAYS OF THE 100 LOTTERY PLAYS. THAT

200 − 92.736 TO 200 + 92.736

#3. SUPPOSE $P(OIL) = .2$ $P(+ \mid_{IF} OIL) = .9$ $P(+ \mid_{IF} OIL^c) = .3$

SUPPOSE COSTS 200 TO DRILL
" 80 TO TEST
GROSS FROM OIL IS 900

$P(+ \mid_{IF} OIL)$



NET POLICY II

$OIL\ +\quad .2(.9) = .18$ $-80-200+900 = 620$

$OIL\ -\quad .2(.1) = .02$ $-80-0+0 = -80$

$OIL^C\ +\quad .8(.3) = .24$ $-80-200+0$ -280

$OIL^C\ -\quad .8(.7) = .56$ $-80-0+0 = -80$

POLICY I
"JUST DRILL"

NET

.2 OIL: $-200+900 = 700$

.8 OIL^C: $-200+0 = -200$

$$E(NET) = 700(.2) - 200(.8) = 140 - 160 = -20$$

POLICY II
"TEST BUT ONLY DRILL IF TEST IS +"

$$E(NET\ II) = 620(.18) - 80(.02) - 280(.24) - 80(.56) = -2$$

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