STT 200 1-27-10a 5:30 p.m. CH 16 & 17

Note Title 1/27/2010

CH 17 PROBABILITY MODELS:

NORMAL
BERNOULLI TRIALS (DEFINITION)
BINOMIAL (ONLY ITS DEFINITION AND NORMAL APPROX)
POISSON (ONLY ITS DEFINITION AND NORMAL APPROX)

SEE EXERCISES 13-17 (NORMAL APPROX) IN RECITATION ASSIGNMENT 2-2-10
18-22 BERNOULLI TRIALS + BINOMIAL IN COIN TOSSES
23-27 SAME AS #18-22 IN TOSSES OF A DIE
28-31 POISSON AS APPROXIMATED BY NORMAL.

I will go over 13-31 today.

13-17. LOTTERY 1: RANDOM VARIABLE X $\quad$ $EX = 17 = \sum_x x\, p(x)$

AND ALSO, $EY = 30$ Variance $Y = 9$ $\quad$ Variance $X = 4$

$= \sum_x (x - EX)^2 p(x)$

OFFER: X, Y INDEP. COSTS 2 for X, 3 for Y

BONUS $X \to 1.4X$.

$$R = \text{NET RETURN} = 1.4X - 2 + Y - 3 = 1.4X + Y - 5$$

$$ER = E(1.4X + Y - 5) \underset{\text{ALWAYS.}}{=} 1.4\,EX + EY - 5$$

$$= 1.4(17) + 30 - 5$$

$$\text{Variance of } R \overset{\text{INDEPENDENCE}}{=} \underline{Var(1.4X)} + Var\,Y + \cancel{Var(-5)} = (1.4)^2 Var X + Var Y$$

$$E(1.4X)^2 - \underline{(E\,1.4X)}^2$$

$$E(1.4^2 X^2) - (1.4\,EX)^2$$

$$1.4^2\, Var X$$

$$E(aX+bY+c) \overset{\text{ALWAYS}}{=} aEX+bEY+c$$

$$Var(aX+bY+c) \underset{\text{INDEP}}{\overset{\text{IF}}{=}} a^2 VarX + b^2 VarY + 0$$

eg $Var(X+3) = E\left((X+3)-E(X+3)\right)^2$

$VarX \quad - \quad E\left(X+3-(EX+3)\right)^2$

13. $R = 1.4X + Y - 5$

14. $ER = 1.4(17) + 30 - 5$

15. $VarR = 1.4^2(4) + 30 + 0$

16. $SD\, of\, R = \sqrt{VarR}$

17. IF X IS "NORMAL"
AND Y IS "NORMAL"
$\underset{\text{INDEP}}{\Longrightarrow}$ R IS "NORMAL"

IF SO THEN

Ch 17. 18-22 ON REC. 2-2-10.

TOSS FAIR COIN $\frac{1}{0}$ $\frac{H}{T}$

"SUCCESS" "FAILURE" — CONTEMPLATE A SEQUENCE OF n INDEPENDENT BERNOULLI TRIALS:

eg COIN TOSSES: H H T T T H T T T T ... T $n = 100$

DIE  ← ace

TOTAL $= X_1 + \cdots + X_n$ = # OF SUCCESSES IN n INDEP TRIALS.

(HAS A BINOMIAL DISTRIBUTION

BERNOULLI TRIALS ARE n INDEP TRIALS EACH OF WHICH IS "SUCCESS" OR "FAILURE" AND THE CHANCE OF SUCCESS IS THE SAME FROM TRIAL TO TRIAL.

$X_i = \begin{cases} 1 & \text{ITH SAMPLE PERSON IS FEMALE} \\ 0 & \text{IF MALE} \end{cases}$

TOTAL $= X_1 + \cdots + X_n$ = # FEMALES.

$E(\text{TOTAL}) = n\,E X_1$
$= np$

$\boxed{EX = 1p + 0q}$

n # TRIALS
p = PR SUCCESS.

BINOMIAL HAS MEAN np.

$\text{Var}(\text{TOTAL}) = n \,\text{Var} X$
$= npq$

$\text{Var} X = EX^2 - (EX)^2$
$= EX - (EX)^2$
$= p - p^2 = pq$

18. 100 TOSSES OF FAIR COIN. $n = 100$ $p = .5$

LET Y = TOTAL # H IN 100 TOSSES.

$y = 0, 1, 2 \ldots, 100$

$P(Y=y)$ FORMULA — NO INTERESTED NOW —

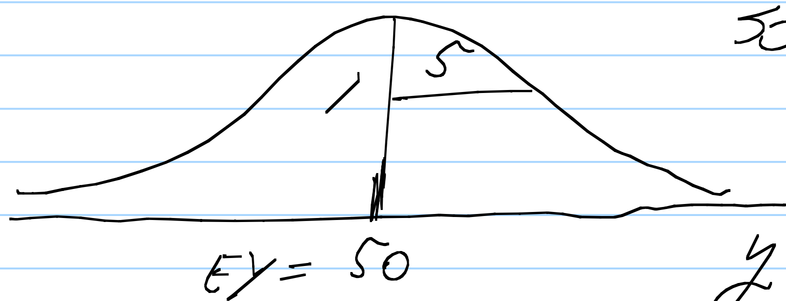
$EY = np = 100\left(\frac{1}{2}\right) = 50$

TOTAL # H IN 100 TOSSES.

$\text{Var}\, Y = np\overset{1-p}{q} = 100 \cdot \frac{1}{2} \cdot \frac{1}{2} = 25$

$SD\, Y = \sqrt{25} = 5$

~DIST OF



SO P(GET BETWEEN 40 + 60 H) ~.95

23–27. COUNTING TOTAL # OF "ACES" ⚀ IN 100 INDEP ROLLS OF A DIE

1 ⚀ SUCCESS
NOT FAILURE

$n = 100 \quad p = 1/6$

$$E\,Y_{(TOTAL)} = np = 100\left(\tfrac{1}{6}\right)$$

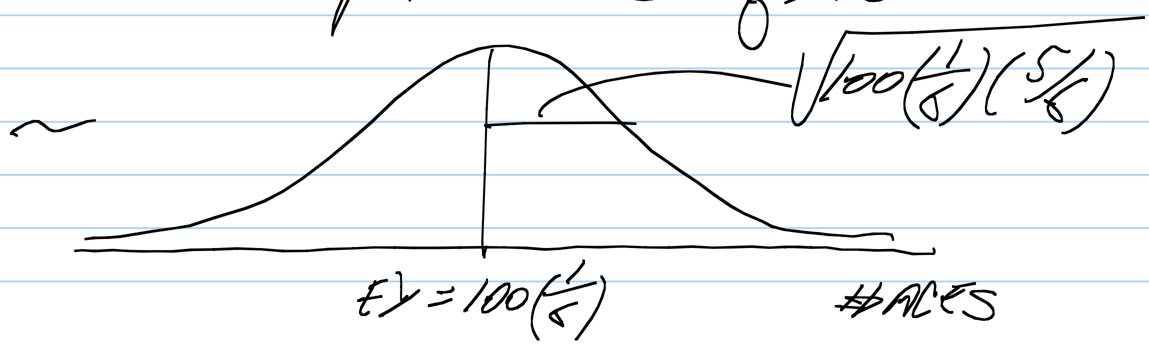
$$\mathrm{Var}\,Y = npq = 100\left(\tfrac{1}{6}\right)\left(\tfrac{5}{6}\right) \Rightarrow SD\,Y = \sqrt{100\left(\tfrac{1}{6}\right)\left(\tfrac{5}{6}\right)}$$

23. $n = 100 \quad p = \frac{1}{6}$

24. $\mathrm{Var}\,X \quad SD\,X$

NOTE: NORMAL APPROX OF BINOMIAL RUNS INTO TROUBLE IF $p \sim 0$ or $p \sim 1$

25: VERIFY TEXT'S CONDITION THAT $np \geq 10$ and $nq \geq 10$

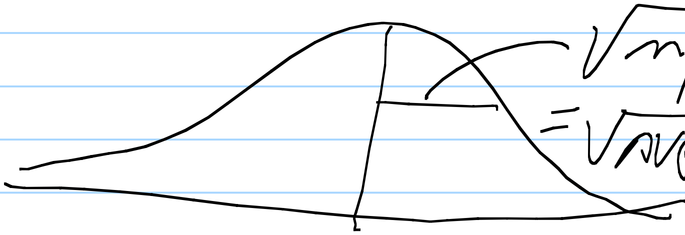


28-31. POISSON (BINOMIAL for $n \sim \infty$, $p \sim 0$)

$$P(x) = e^{-np}(np)^2 / x! \qquad x = 0, 1, \ldots, n \qquad (0! = 1)$$

IF (RULE OF THUMB) $np \geq 3$ (ie $E(\#\text{SUCCESSES}) \geq 3$) NOT FOR NOW

THEN $\sim$



$\sqrt{npq} = \sqrt{\text{AVG}\#}$

$p \sim 0$

$q \sim 1$

np ie AVG#

SUPPOSE WE AVG (FROM PAST EXPERIENCE) AROUND 4.2 PERSONS HIT BY LIGHTNING.

28. $EX \cong 4.7$

n? p?

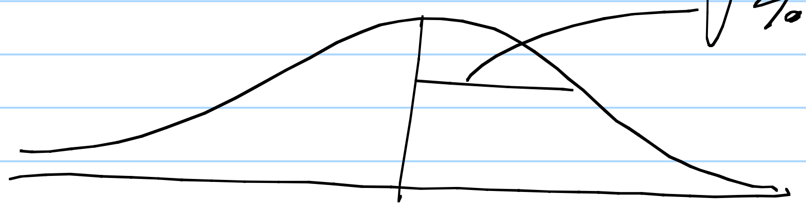
29. $SD\,X \cong \sqrt{4.7}$ VERY SPECIAL TO POISSON.

POISSON

$np \sim 4.7$

30. $4.7 \geq 3$ so

$\sqrt{4.7}$ POISSON



4.7

#HITS BY LIGHTNING

31. $4.7 \pm 2\sqrt{4.7}$ FOR 95%