STT 200 3pm. 2-1-10

Note Title 2/1/2010

REVIEW FOR EXAM 1 (PART)

ADDITIONAL COVERAGE OF RECITATION ASSIGNMENT. DUE TOMORROW.

1-12. RANDOM VARIABLE X

x	$p(x)$	$x\,p(x)$	$(x-(-0.7))^2\,p(x)$	$x^2\,p(x)$
0	0.2	$0\,(.2)=0$	$(0-(-0.7))^2\ 0.2$	$0^2\,(.2)$
1	0.3	$1\,(.3)=.3$	$(1-(-0.7))^2\ 0.3$	$1^2\,(.3)$
-2	0.5	$-2\,(.5)=-1$	$(-2-(-0.7))^2\ 0.5$	$(-2)^2\,(.5)$
		$EX=-0.7$	$\text{Var}\,X \underset{\text{DEF}}{=} 1.81$	$EX^2=2.3$

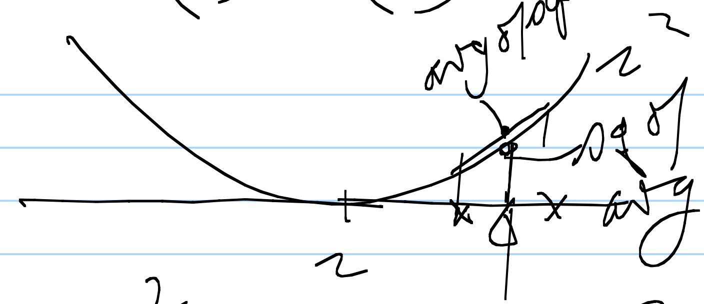
-0.7 from EX

← READ OFF →

also, $\underbrace{\operatorname{Var} X = E(X^2) - (EX)^2}$

— MUST KNOW —

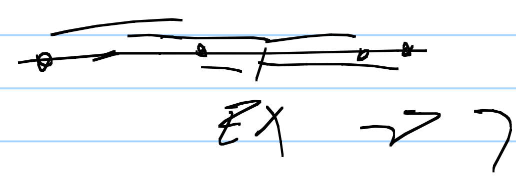
$E(X^2) \ge (EX)^2$



1. $EX = -0.7 \qquad \sum_x x\,p(x)$
2. $\operatorname{Var} X = 1.81$
3. $\operatorname{Var} X \overset{\text{also}}{=} E(X^2) - (EX)^2 = 2.3 - (-0.7)^2$
   $= 2.3 - .49 = 1.81$
4. $E(3X+7) \underset{\text{LINEARITY OF } E}{\overset{\text{ALWAYS}}{=}} 3EX + 7$
   $= 3(-0.7) + 7$
5. $E(3X - X - 6) = 3EX - EX - 6 = 2EX - 6$

Same as $E(2X-6) = 2EX - 6$

6. $\operatorname{Var}(3X + \cancel{7}) = \operatorname{Var}(3X) = 3^2 \operatorname{Var} X$



7. $Var(3X - X + 7)$

$= Var(2X + \cancel{7})$

$= 4 \; Var\, X = 4(1.81)$

$Var(aX) = E(a^2X^2) - (E(aX))^2$

$= E a^2 X^2 - (aEX)^2$

~~$\underbrace{Var\,3X} + Var(-X) + Var\,7$~~

~~$3^2 Var\,X + (-1)^2 Var\,X$~~ 0

~~$9(1.81)$~~

ALL WRONG

$3X$ IS <u>NOT</u> INDEP OF $-X$

8. additionally, Y is a RANDOM variable

$E(X - 2Y + 4) \underset{\text{ALWAYS}}{=} EX - 2EY + 4$

RULE:

$Var(aX + \cancel{b}) = a^2 Var\,X$

9. Spse $Var\,Y = 2$ $Var\,X = 1.81$ (from above)

assume X, Y INDEP

? $Var(5X - Y + \cancel{4}) \overset{\text{INDEP}}{=} 25\, Var X + 2$

$= 25 \quad 1.81 + 2$

10. $SD\ X = \underset{\text{SIGMA}}{\sigma} = \sigma_{X} = \sqrt{Var\ X}$

NAME OF THE RANDOM VARIABLE WHOSE SD IS REFERRED TO.

SO $\sigma_X = \sqrt{1.81}$

11. SD of $(3X - 2, X + \cancel{4}) = SD(X) = \sqrt{1.81}$

12. REFER TO #9 ? $SD(5X - Y + \cancel{4}) = \sqrt{Var(5X - Y)}$

$\underset{\text{IF INDEP}}{=} \sqrt{25(1.81) + 2}$

RULE

$\sigma_{X+Y} \underset{\text{INDEP}}{=} \sqrt{\sigma_X^2 + \sigma_Y^2}$ PYTHAGORAS' FORMULA

CHANGE FROM THE RES. ASSIGNMENT TO

PARADOX OF THE FALSE POSITIVE.

	DISEASED	NOT	
+	1	20	21
−	0	979	979
			1000

$P(+|Dis) = 1 \quad P(+|Dis^c) = \frac{20}{999}$

$\sim .002$

$$P(Dis \,|\, +) = P(Dis\,+)/P(+) = \frac{1}{21} \sim .05$$

LOOKS LIKE A GOOD TEST!

BUT OVERWHELMED BY TOO MANY NON-DISEASED PEOPLE TAKING THE TEST.

A RELATED EXAMPLE, MASSIVE UN-TARGETED PROMOTIONS CAN ACTUALLY REDUCE SALES.

AS TO EXAM 1

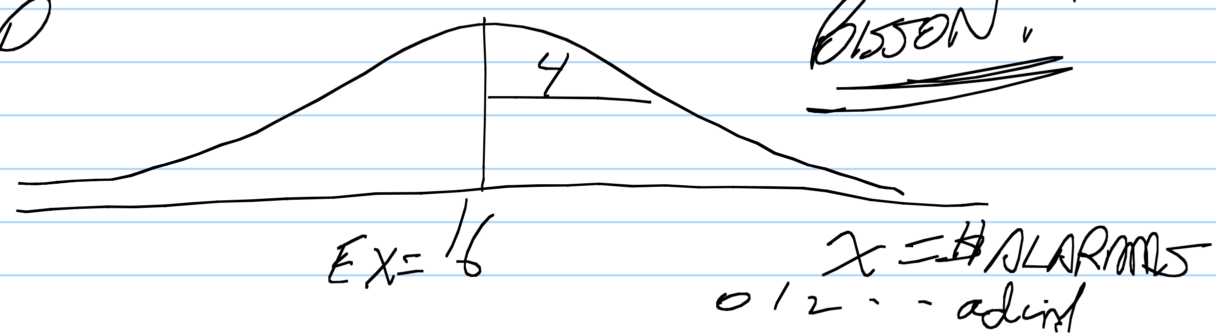
1. WE AVG AROUND 16 FIRE ALARMS PER DAY.
SUPPOSE POISSON DIST^N FOR X = # ALARMS
TOMORROW. TOLD $EX \sim 16$.

POISSON + $EX = 16$ $\Rightarrow$ $\left[\sigma_X \overset{\text{POISSON}}{=} \sqrt{\mathrm{Var}\, X} = \sqrt{16} = 4 \right]$

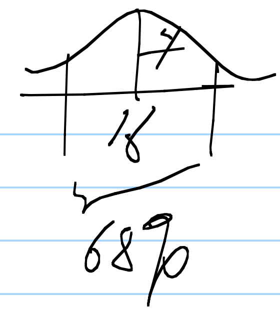
ALSO IF $EX \geq 3$ BELL CURVE APPROX IS
RECOMMENDED

POISSON !!

$\approx$



SO $P(X \text{ IN } [16-4\ ,\ 16+4]) \sim 0.68$



$P(X \text{ IN } [16-8\ ,\ 16+8]) \sim 0.95$

2. <u>BINOMIAL</u> SUPPOSE 75% OF VOTERS FAVOR PROPOSAL.
RANDOM SAMPLE $n = 400$ VOTERS.

X (RANDOM) = # OF THE 400 WHO FAVOR PROPOSAL

$x = 0, 1, \ldots, 400$

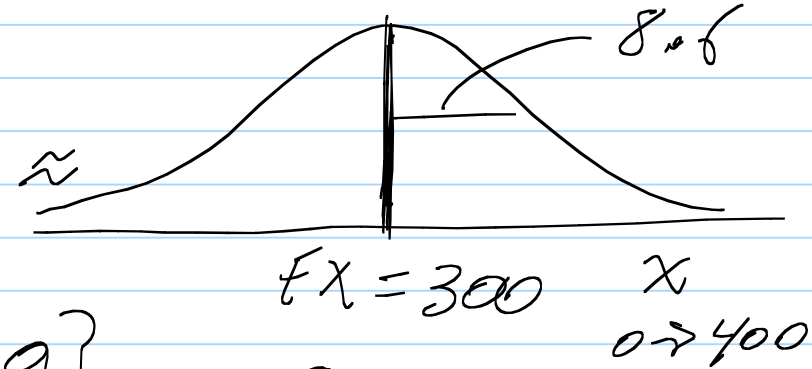
$$EX = np = 400(.75) = 300$$

$$\text{Var } X = np\underbrace{q}_{(1-p)} = 400 \cdot .25 \cdot .75 = 75$$

$$\sigma_X = \sqrt{npq} = \sqrt{75} = 8.6$$

$300 \geq 10$

$400 - 300 \geq 10$ $\Rightarrow$ NORMAL APPROX OK (THIS CLASS)

$\approx$ 

$[300 - \cancel{8.6}\,9,\ 300 + 9]$ 68%

$2(8.6) \approx 17$ $[300 - 17,\ 300 + 17]$ 95%.

(NOTE: BINOMIAL Var $X = npq$ | IF $n \sim \infty$, $p \sim 0$, $q \sim 1$ BECOMES: $np1 = EX$

3. LOTTERY PRODUCES RANDOM VARIABLE Y.

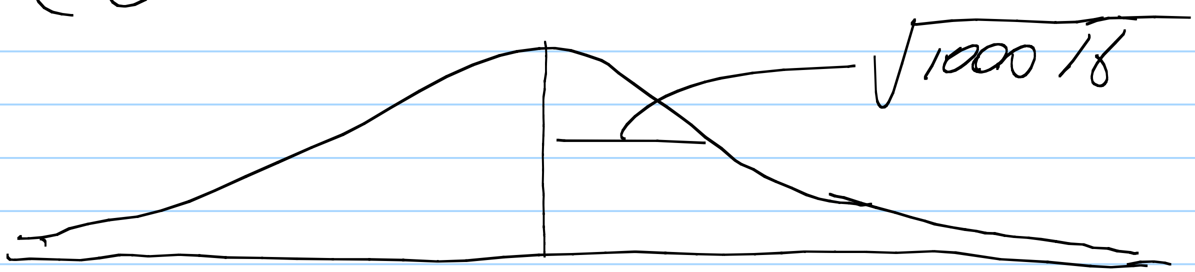
$$E(Y) = \$2 \qquad \mathrm{Var}\, Y = \$^2\, 16 \qquad \sigma_X = \$4$$

MANY INDEP PLAYS $\quad TOT = Y_1 + Y_2 + \cdots + Y_{1000}$

$$E(TOT) = n\, E\, Y_1 = 1000\,(2) = \$2000$$

$$\mathrm{Var}(TOT) \overset{INDEP}{=} n\, \mathrm{Var}\, Y_1 = 1000\ 16$$

$\approx$



$$E\, TOT = 1000\,(2) = 2000$$

| ← | 68% | → | 95%