STT 200 5:30 2-10-10a

Note Title 2/10/2010

Ch 18, Ch 19, CLASSROOM EXERCISE.

BEGAN CH 18 BERNOULLI TRIALS $\begin{matrix} Y & p \\ N & q=1-p \end{matrix}$ n INDEP TRIALS.

BINOMIAL: X = # of "Y" in n trials.

VALUES POSSIBLE $x = 0, 1, \ldots, n$ (# Y in n)

BOOK $\Rightarrow$ $P(X = x) = \dfrac{n!}{x!(n-x)!} p^x q^{n-x}$ $\quad x = 0, 1, \ldots, n$

RANDOM — A VALUE IN $0, 1, \ldots, n$

e.g. $3! = 3(2)(1)$ $\quad 0! \overset{DEF}{=} 1$ $\quad P(0) = q^n$ $\quad \leftarrow$ NO

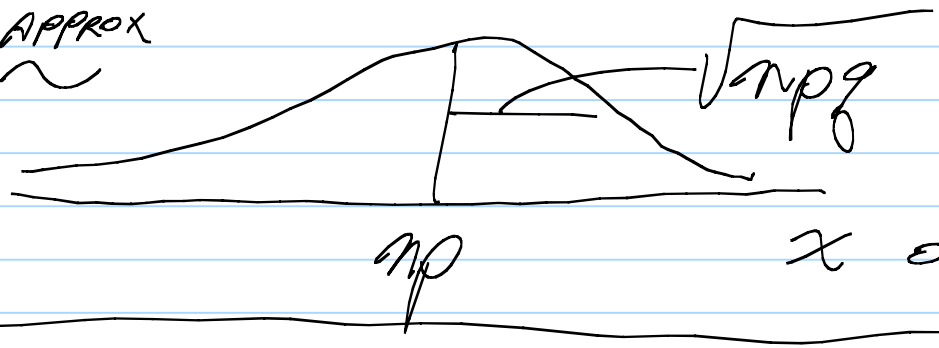
$N N N \cdots N$ (n)

$1 \; p^0 q^{n-0}$

NICE THAT WE'VE NORMAL APPROX.

IF $n \to \infty$, $p \neq 0$ $p \neq 1$ (CONDITIONS $np \geq 10, nq \geq 10$).

THEN DIST OF $X \overset{\text{APPROX}}{\sim}$



IN MANY SITUATIONS WE FOCUS ON $\hat{p} = \frac{X}{n} = \frac{\#\text{'s in } n}{n}$

eg IF GET 325 REP IN RANDOMM SAMPLE OF $n = 600$

THEN $\hat{p} = \frac{325}{600}$.

FROM ABOVE, $E(\hat{p}/n) = E\hat{p}/n = \frac{np}{n} = p$ RULES

$$\sigma_{\hat{p}/n} = \frac{1}{n}\sigma_{\hat{p}} = \frac{1}{n}\sqrt{npq} = \sqrt{pq/n}$$ RULES

DIST^N OF $\hat{p}$ $\approx$

SO FROM THIS $P(\hat{p}$ IN INTERVAL $p \pm 1\sqrt{pq/n}) \approx .68$

SO TOO $P(\hat{p}$ IN $p \pm 2\sqrt{pq/n}) \approx .95$ NOTE: $\sigma_{\hat{p}} = \sqrt{pq/n}$.

$$\Rightarrow P\left(p \text{ IN } \left(\hat{p} \pm 1\sqrt{pq/n}\right)\right) \approx .68$$

↑ THIS IS TANTALIZING - IF ONLY WE KNEW $\sqrt{pq/n}$ WE'D HAVE A RANGE AROUND $\hat{p}$ IN WHICH p MIGHT BE FOUND.

HERE IS THE REMARKABLE FACT:

$$P\left(p \text{ COVERED BY } \hat{p} \pm 1\sqrt{\hat{p}\hat{q}/n}\right) \sim .68$$

WALK THROUGH:

SAMPLE AT RANDOM $n = 600$ VOTERS.

FINDING $\hat{p} = \frac{320}{600}$ (Y = REP) $= x/n$.

MY POINT ESTIMATE OF p COULD BE $\hat{p} = \frac{320}{600} = .54$

$P(\hat{p}$ ACTUALLY $= p) \sim 0$.

FAT GUESS $\hat{p} \pm 1\sqrt{\hat{p}\hat{q}/n} = .54 \pm 1\sqrt{\frac{.54 \cdot .46}{600}}$

$= .54 \pm .020$ or $[.52, .56]$

CLAIM: ~68% OF SAMPLES YIELD AN INTERVAL COVERING TRUE p.

for $np \geq 10$ ~~$nq \geq 10$~~

$\sim$ 95% OF SAMPLES OF $n = 600$ YIELD AN INTERVAL $\hat{p} \pm 2\sqrt{\hat{p}\hat{q}/n} = [.50, .58]$ (IN THIS CASE) COVERING TRUE p.

EXERCISE 68% CONFIDENCE INTERVAL: $\hat{p} \pm 1\sqrt{\hat{p}\hat{q}/n}$

"NOT 2" — ACTUALLY $\sigma_{\hat{p}}$

$p = .5$] 100
$.6$] 100
$.4$] 100

$.5$] 60
$.6$] 60
$.4$] 60

$.5$] 30
$.6$] 30
$.4$] 30

COVER NOT
q $\sqrt{}$
// $\sqrt{3}$

[,]

? $\nearrow$ $.5$