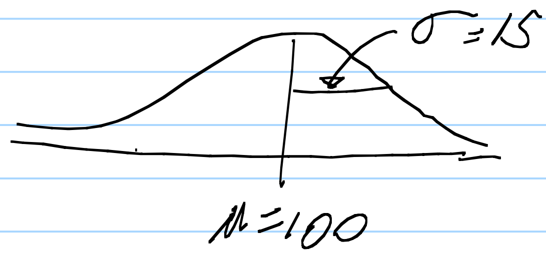
Note Title 2/8/2010

STT 200 3pm 2-8-10 Ch 18 (PART CH 19)

TODAY: COVER YOUR RECITATION ASSIGNMENT DUE TOMORROW AND INITIATE SOME DISCUSSION OF "CONFIDENCE INTERVAL FOR p" INTRODUCED IN CH 19 (BUILDS ON 18).

BELL CURVE



$\mu_{IQ} = 100$ "mu" μ

$\sigma_{IQ} = 15$ "sigma" σ

PRECISE IDEAL

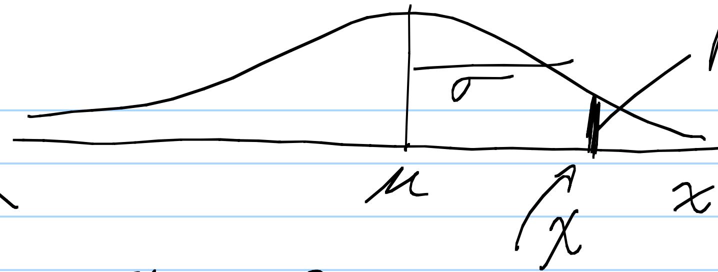
$$p(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{-(x-\mu)^2/(2\sigma^2)}$$

$x = IQ$

$\pi \sim 3.14159\ldots$

$e \sim 2.7\underbrace{1819}\underbrace{1819}\ldots$

IDEAL IQ CURVE

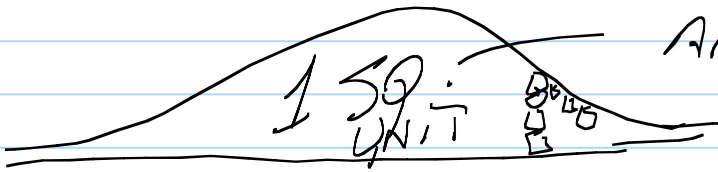


$$p(x) = \left(\frac{1}{15\sqrt{2\pi}}\right) e^{-\frac{(x-100)^2}{2\cdot 15^2}}$$

x = IQ SCORE

PROPERTIES:

[REGARDLESS OF μ, σ OF A NORMAL CURVE, THE AREA UNDER THE CURVE IS 1]

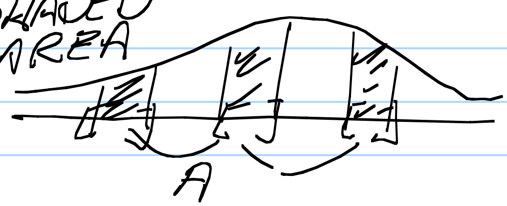


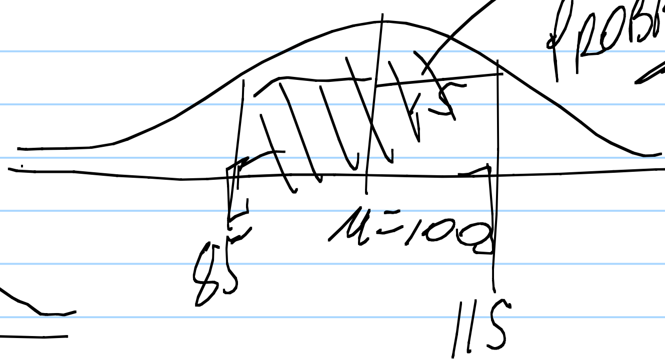
AREA IS 1.0

CONNECT WITH PROBABILITY:

$P(IQ \in A)$ = SHADED AREA

"IN"



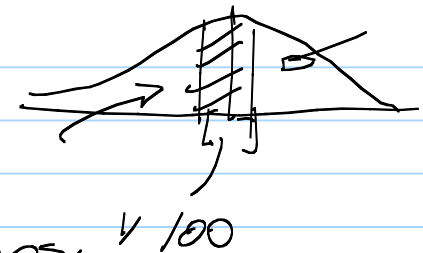


.68 PROBABILITY

ONE IMPORTANT THING: $P(IQ = \text{THE})$

$P(IQ = 100) <$

MUST BE ZERO!



CURIOSITY OF SO-CALLED "CONTINUOUS MODELS"

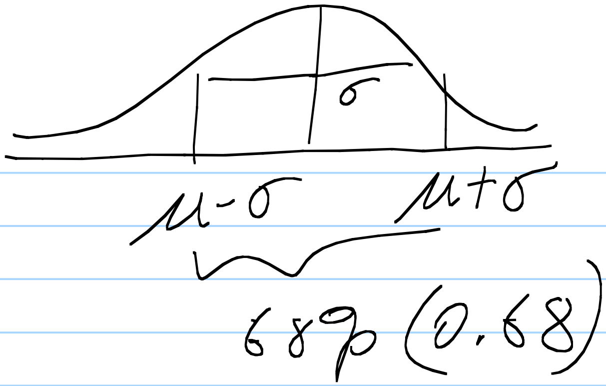
IS $P(X = x) = 0$

$\uparrow$ ANY PARTICULAR x

DENSITY $p(x) =$ "LOCAL RATE OF ACCUMULATION OF PROBABILITY."

A MOST VALUABLE PROPERTY OF NORMAL MODELS.

"ALL NORMALS CAPTURE SAME PROBABILITY WHEN INTERVALS ARE IN σ UNITS FROM μ"



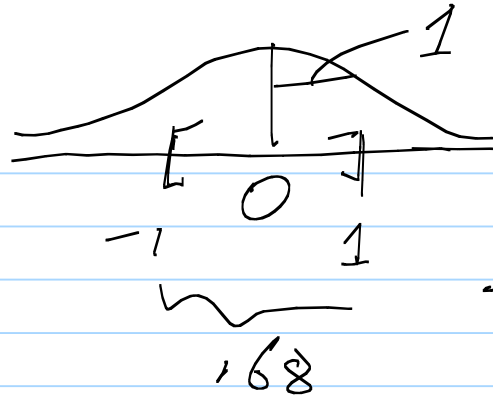


TABLE AREAS UNDER SO-CALLED STANDARD NORMAL "z-TABLE"

RANDOM VARIABLE X, μ_X, σ_X NORMAL

X

"Say my IQ is 121" $\frac{121-100}{15}$ is my z-SCORE OF IQ

STANDARD SCORE

$$P(\underset{\text{"IN"}}{IQ \in} [100-15, 100+15]) = P\left(Z \in \left[\frac{85-100}{15}, \frac{115-100}{15}\right]\right)$$

85 $\quad$ 115

INDEED

$$\overset{ANS}{=} P(Z \in [-1,1]) \qquad = P(Z \in [-1, 1])$$

2a. $IQ = 115 \Rightarrow$ z-SCORE IS $\dfrac{115 - \overset{\leftarrow \mu \text{ OF } IQ}{100}}{15 \leftarrow \sigma \text{ OF } IQ} = 1$

2b. $IQ = 100 \Rightarrow \dfrac{100 - 100 \leftarrow \mu}{15 \leftarrow \sigma} = 0$

2c. $IQ = 130 \Rightarrow z = \dfrac{130-100}{15} = 2$

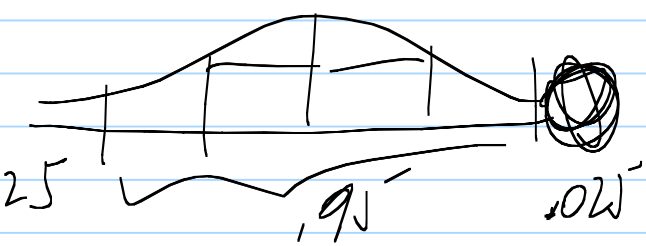
REMARK: $IQ = 128.3 \Rightarrow z = \dfrac{128.3-100}{15}$

2d. GIVE $z = 1 \Rightarrow IQ = 100 + z\,15 = 100 + (1)15$
$= 115$

2e. If $z = -2 \Rightarrow IQ = 100 - (2)15 = 70$.

3. $P(IQ > 130) = P\left(Z > \frac{130-100}{15}\right) = P(Z > 2)$

$\frac{IQ-100}{15}$ $\quad > .025$

a. 

b. $P(85 < IQ < 115) = P\left(\frac{85-100}{15} < Z < \frac{115-100}{15}\right)$

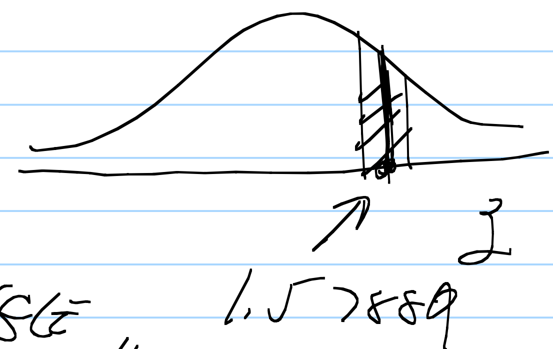
$= P(-1 < Z < 1) = .68$

c. $P(|IQ - \overset{\mu}{100}| < \overset{\sigma}{15}) = P(|Z| < 1)$

d. $P(IQ < \underbrace{100}_{\mu}) = P(Z < \underset{=\,\mu}{0}) = .5$

e. $P(70 < IQ < 115) = P(-2 < Z < 1)$

h. $P(Z = 1.57889) = 0$

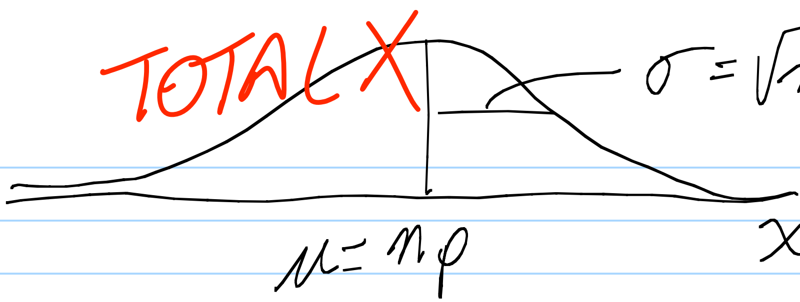


4. RECALL BINOMIAL n INDEP TRIALS.
p = CHANCE ANY SINGLE TRIAL IS A "SUCCESS"

LET X = # SUCCESSES IN n TRIALS.
DISCRETE $x = 0, 1, \ldots, n$.

$np \geq 10$

$nq \geq 10$

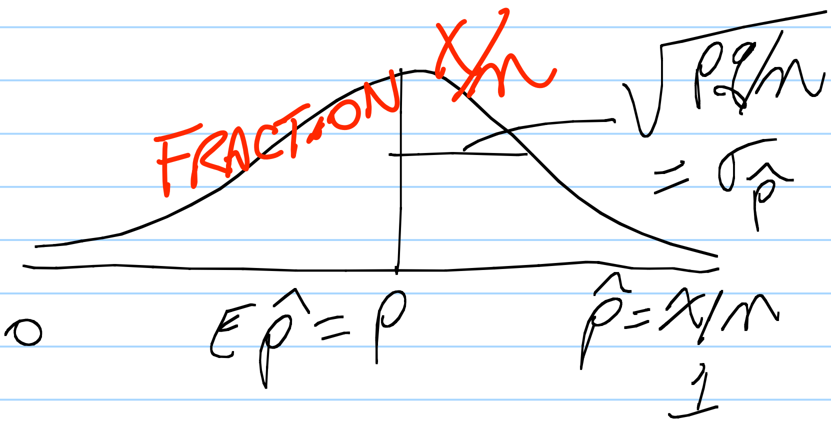


$EX = np$

$\sqrt{\mathrm{Var}\,X} = \sigma_X = \sqrt{npq}$

CHANGE THE FOCUS TO $\hat{p} = \frac{X}{n}$ eg $\frac{48}{100} \leftarrow$ # H IN 100 TOSSES

$\hat{p} = 0.48$ OUR ESTIMATE OF p



$$E\frac{X}{n} = \frac{1}{n}EX = \frac{1}{\cancel{n}}\cancel{n}p = p$$

$$\sigma_{X/n} = \frac{1}{n}\sigma_X = \frac{\sqrt{npq}}{n} = \sqrt{\frac{pq}{n}}$$

PLAY "WHAT IF"

POPULATION OF VOTERS.

SAMPLE $n = 400$

SUPPOSE 53% OF POPULATION FAVOR PROPOSAL.

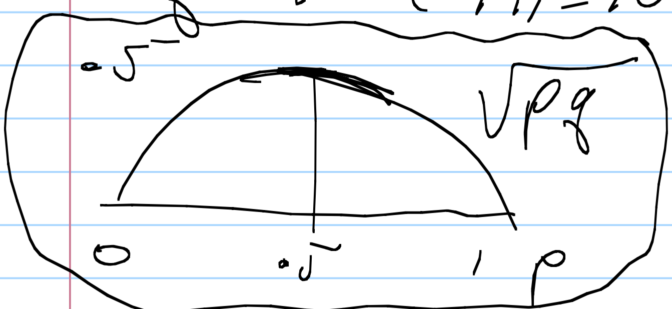
$$\sqrt{.53 \cdot .47 / 400} \sim \tfrac{1}{2}/\sqrt{400} = \tfrac{1}{40}$$

$p = .53 \qquad \hat{p} = \tfrac{1}{n}$

$\approx$

$np = 400(.53) \geq 10$

$nq = 400(.47) \geq 10$



68% PROBY $\hat{p} \in \left[.53 - \tfrac{1}{40}, .53 + \tfrac{1}{40}\right]$

95% " $\hat{p} \in \left[.53 - \tfrac{2}{40}, .53 + \tfrac{2}{40}\right]$