STT 200 5:30pm 2-8-10

Note Title 2/8/2010

Ch 18 NORMAL, NORMAL APPROX OF BINOMIAL $np, \sqrt{npq}$ (μ, σ)

NORMAL APPROX OF $x/n = \hat{p}$ $\quad p, \sqrt{pq/n}$ (μ, σ)

STANDARD (or z) SCORES. $z = \frac{x-\mu}{\sigma}$, $x = \mu + \sigma z$

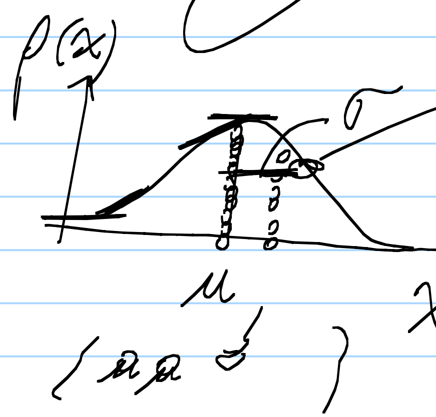
I plan to go over the recitation assignment ~~today~~.

NORMAL DIST^N ON $(-\infty, \infty)$

$$p(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\pi \sim 3.14159\ldots$

$e \sim 2.718281828\ldots$



BINPOT

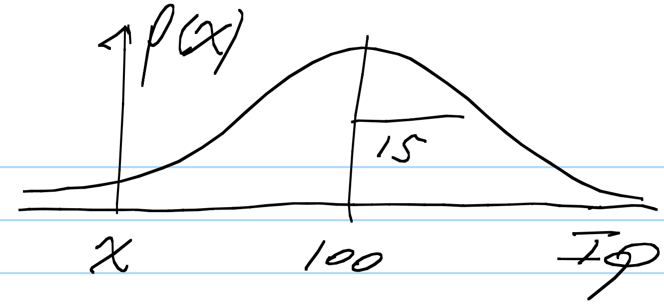
INFLECTION

SLOPE

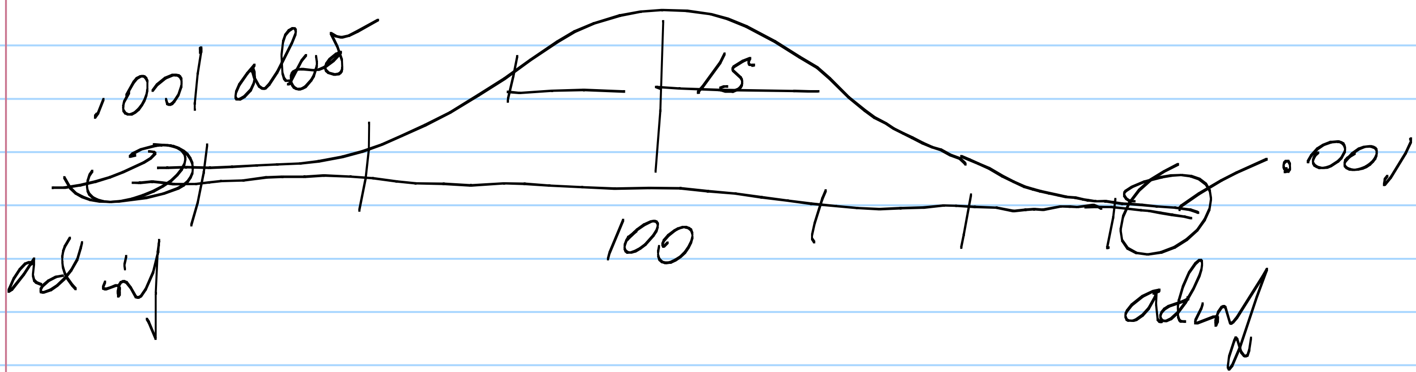
STALLS

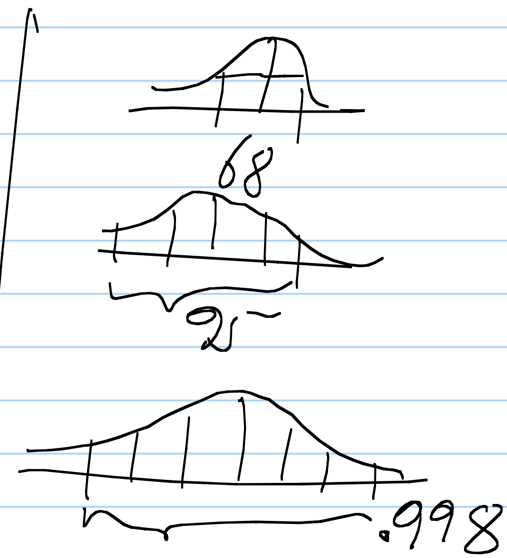
EXAMPLE IQ $\mu_{IQ} = 100$, $\sigma_{IQ} = 15$

$$p(x) = \left(\frac{1}{15\sqrt{2\pi}}\right) e^{-(x-100)^2/(2\,15^2)}$$



PROBY OUTSIDE $\pm 3.09\,\sigma$ UNITS FROM μ IS (ANY NORMAL) $\sim .002$





"ALL NORMALS ARE ALIKE IN AREAS CAPTURED BY INTERVALS EXPRESSED IN σ UNITS FROM μ!!

CONCEPT OF STANDARD SCORE: RANDOM VARIABLE X, $\underset{\text{mu}}{\mu_X}$, $\underset{\text{sigma}}{\sigma_X}$

SAY MY IQ IS 118. STANDARD SCORE $\dfrac{118-100}{15}$

z (STANDARD SCORE, z-SCORE) $\overset{DEF}{=} (x-\mu)/\sigma$

z IS # OF SD REMOVED FROM μ.

SO $x = \mu + z\sigma$ — eg IF MY z-SCORE AT BP IS 3.11 THEN MY BP IS

$$x_{BP} = \mu_{BP} + 3.11\,\sigma_{BP}.$$

z DISTRIBUTION (TABLED) IS NORMAL WITH $\mu_z = 0$, $\sigma_z = 1$

2a. IQ PERSON HAS IQ = 115 $\Rightarrow z = \frac{x-100}{15} = \frac{115-100}{15} = 1$

2b. IQ = 100 $\Rightarrow z = \frac{100-100}{15} = 0$

2c. IQ = 130 $\Rightarrow z = \frac{130-100}{15} = 2$

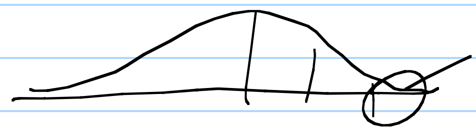
2d. If $z = 1 \Rightarrow x = \mu + z\sigma = 100 + 1\,(15) = 115$

2e. If $z = -2 \Rightarrow x = \mu - 2\sigma = 100 - 2(15) = 70.$

METHOD: $P(X < 17.2) = P\left(Z < \frac{17.2 - \mu_x}{\sigma_x}\right)$ TABLED

IF X IS NORMAL OR APPROXIMATELY SO.

3a. $P(IQ > 130) = P\left(Z > \frac{130-100}{15}\right) = P(Z > 2) \approx .025$



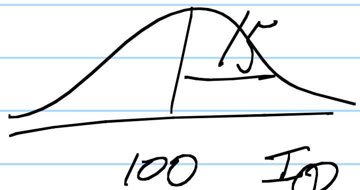
$.05/2 = .025$

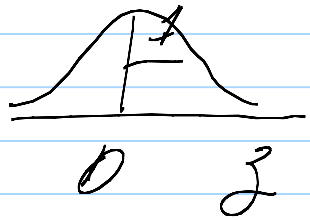
3b. $P(85 < IQ < 115) = P(-1 < Z < 1) \approx 0.68$

$-1 = \frac{85-100}{15}$

3c. $P(|IQ-100| < 15) = .68$

3d. $P(IQ < 100) = P(Z < 0) = 0.5$





e. $P(70 < IQ < 115) = P(-2 < Z < 1)$

$\mu - 2\sigma$

$100 - 2(15)$

$\mu + \sigma$

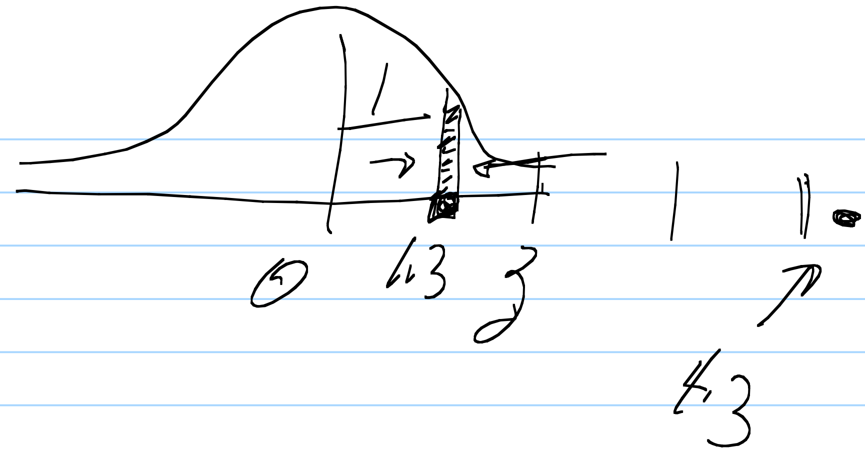
$100 + 15$

$\mu + 3.09(15)$

? g. $P(Z > 3.09) \sim 0.001$ (TABLE) $\Rightarrow P(IQ > 145) = P(Z > 3.09)$

$= .001$

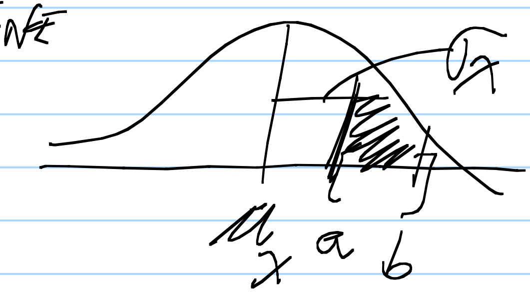
h. $P(Z = \cancel{4.3}\ 1.3) = 0$

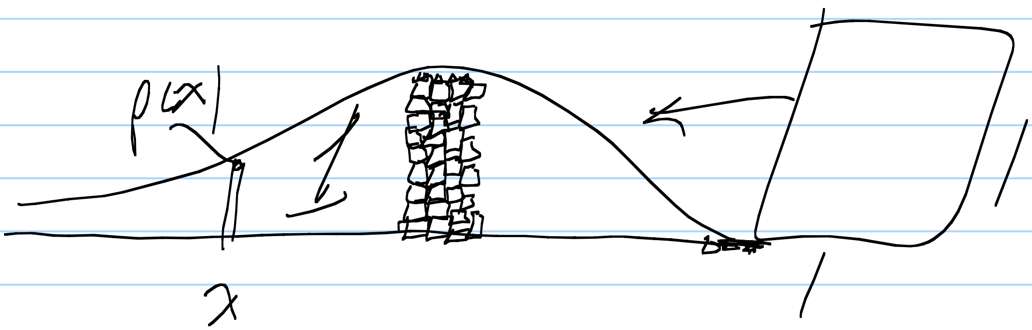


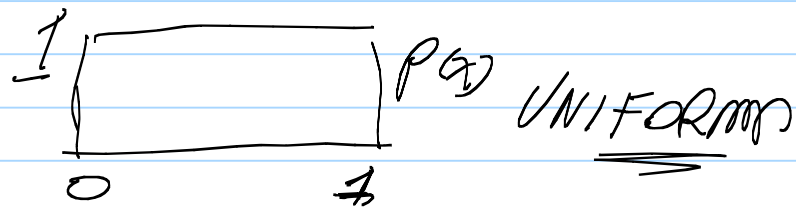
area under each normal $p(x)$

is 1.0

$P(X \text{ in } [a,b]) \overset{\text{DEFINE}}{=}$







IN CONTINUOUS PROBABILITY MODELS:

$f(x)$ IS THE "LOCAL RATE OF ACCUMULATION OF PROBABILITY"

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4. NORMAL APPROX OF BINOMIAL

X = RANDOM VARIABLE COUNT OF # SUCCESSES IN n INDEP TRIALS WITH PR p OF SUCCESS ON EACH TRIAL.

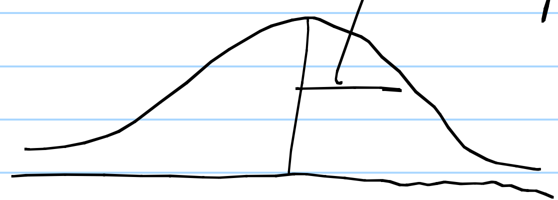
DIST OF X

IF $np \geq 10$

$nq \geq 10$

$\approx$

$\sigma_X = \sqrt{npq}$



$\mu_X = np$

CH18 — LOOK AT: $\hat{p} = X/n$ eg $\frac{48}{100}$ ~ HEADS, ~ TOSSES $\hat{p} = \frac{48}{100} = 0.48$

RANDOM.

$EX = np$

? $E(X/n) = \frac{1}{n} EX = \frac{1}{\cancel{n}} \cancel{n} p = p$

? $\sigma_{X/n} = \frac{1}{n} \sigma_X = \frac{1}{n}\sqrt{npq}$

$= \sqrt{pq/n}$

SO: $\tilde{\sim}$ DIST OF $\hat{p}$

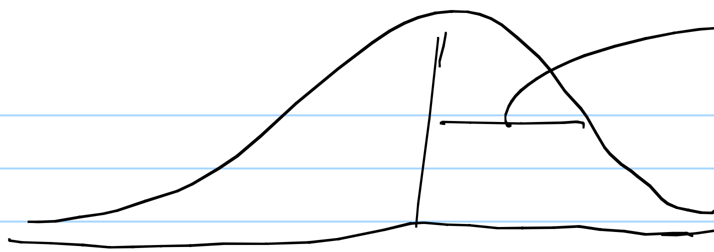
$\sigma_{\hat{p}} = \sqrt{pq/n}$

$E\hat{p} = p$

EXAMPLE #5: $n = 30$ VOTERS SUPPOSE (WHAT IF) $p = .55$ FRAC OF POP DEM

$np \geq 10$ $\approx$

$nq \geq 10$



$\mu_{\hat{p}} = .55$ $\qquad$ $\hat{p} = \frac{x}{30}$

$\sqrt{\frac{.55 \cdot .45}{30}} \approx \frac{\left(\frac{1}{2}\right)}{\sqrt{30}}$

$\sqrt{pq} = \sqrt{\frac{1}{2}\frac{1}{2}} = \frac{1}{2}$