

1. People are equal-probability selected from a population and sorted by sex and gene type. Which one of the tables below exhibits independence between sex and gene type?

- a.
- | | | | |
|--------|----|----|----|
| | AA | Aa | aa |
| male | 60 | 30 | 40 |
| female | 90 | 45 | 30 |
- b.
- | | | | |
|--------|----|----|----|
| | AA | Aa | aa |
| male | 20 | 30 | 10 |
| female | 30 | 45 | 15 |
- c.
- | | | | |
|--------|----|----|----|
| | AA | Aa | aa |
| male | 30 | 30 | 10 |
| female | 20 | 45 | 15 |
- d.
- | | | | |
|--------|----|----|----|
| | AA | Aa | aa |
| male | 20 | 15 | 10 |
| female | 50 | 45 | 20 |
- e.
- | | | | |
|--------|----|----|----|
| | AA | Aa | aa |
| male | 40 | 20 | 20 |
| female | 60 | 90 | 30 |

$$\frac{20}{30} = \frac{30}{45} = \frac{10}{15} = \frac{2}{3}$$

2-4. A plane has two engines. The probability of the left engine failing is 0.002, same as for the right engine failing. Engine failures are independent events.

2. What is the probability that neither engine fails?

- a. $0.002 + 0.002 - 0.002 \cdot 0.002$ b. $0.002 \cdot 0.002$ c. $0.998 \cdot 0.002$
 d. $0.998 \cdot 0.998$ e. 0.994

$$(1 - 0.002) \cdot (1 - 0.002) = 0.998 \cdot 0.998$$

3. What is the probability that (the left engine fails) and (the right does not)?

- a. $0.002 + 0.002 - 0.002 \cdot 0.002$ b. $0.002 \cdot 0.002$ c. $0.998 \cdot 0.002$
 d. $0.998 \cdot 0.998$ e. 0.994

$$(1 - 0.002) \cdot 0.002$$

4. What is the probability that (the left engine fails) or (the right engine fails)?

- a. $0.002 + 0.002 - 0.002 \cdot 0.002$ b. $0.002 \cdot 0.002$ c. $0.998 \cdot 0.002$
 d. $0.998 \cdot 0.998$ e. 0.994

$$\begin{aligned}
 & p(\text{left engine fails}) \\
 & + p(\text{right engine fails}) - p(\text{both fails}) \\
 & = 0.002 + 0.002 - (0.002)^2
 \end{aligned}$$

5-6. A person will be selected at random from all those sorted by sex and gene type in the table below.

	AA	Aa	aa
male	4	8	5
female	6	9	3

5. What is the probability the person does not have gene type AA?

- a. $\frac{13}{19}$ b. $\frac{17}{35}$ c. $\frac{4}{17}$ d. $\frac{25}{35}$ e. $\frac{13}{35}$

$$\frac{35-10}{35}$$

6. What is the conditional probability $P(\text{AA} \mid \text{if male})$, that the selected person has gene type AA if they are male?

- a. $\frac{13}{19}$ b. $\frac{17}{35}$ c. $\frac{4}{17}$ d. $\frac{25}{35}$ e. $\frac{13}{35}$

$$\frac{4}{17}$$

7-10. Suppose that $P(\text{OIL}) = 0.4$, $P(+ \mid \text{if OIL}) = 0.7$, $P(+ \mid \text{if OIL}^c) = 0.2$.

7. $P(- \mid \text{if OIL}) =$

- a. 0.6 b. 0.3 c. 0.8 d. 0.7 e. 0.4

$$\frac{0.4 \times (1-0.7)}{0.4} = 0.3$$

8. $P(+)$ =

- a. 0.6 b. 0.3 c. 0.8 d. 0.7 e. 0.4

$$0.4 \times 0.7 + 0.6 \times 0.2 = 0.4$$

9. $P(\text{OIL} \mid \text{if } +) =$

- a. 0.6 b. 0.3 c. 0.8 d. 0.7 e. 0.4

$$\frac{0.28}{0.4} = 0.7$$

Suppose also that the cost to test is 20, the cost to drill is 80, and the gross return from finding OIL is 700.

10. NET return from policy "test, but only drill if the test is positive" for the contingency

OIL⁻ is equal to:

- a. -100 b. 600 c. 580 d. -20 e. 680

test -20
No drill

11-12. The distribution of IQ is normal with mean 100 and standard dev 15.

11. The probability of an IQ in range [100-15, 100+15] is approximately:

- a. 0.34 b. 0.475 c. 0.11 d. 0.68 e. 0.95

one std away

12. The probability of IQ in range [100+15, 100+30] is approximately:

- a. 0.34 b. 0.475 c. 0.135 d. 0.68 e. 0.95 (draw a picture and work with pieces)

$$\frac{0.95}{2} - \frac{0.68}{2} = 0.135$$

13-17. A lottery has return X which is a random variable with

$$E X = 3 \quad \text{Var } X = 4$$

13. Standard deviation of $X =$

- a. 4 b. 1 c. 7 d. 2 e. 3

14. Variance of $(2X + 7) =$

- a. 1 b. 8 c. 16 d. 53 e. 23

$$4 \times \text{Var } X = 16$$

For 100 independent plays of the lottery define $T = X_1 + \dots + X_{100}$.

15. $E T =$

- a. 30 b. 7 c. 70 d. 300 e. 3

$$100 \times 3 = 300$$

16. $\text{Var } T =$

- a. 400 b. 200 c. 20 d. 10 e. 530

$$4 \times 100 = 400$$

17. Approximate 68% interval for $T =$

- a. [220, 380] b. [280, 320] c. [160, 280] d. [235, 365] e. [260, 340]

$$300 \pm \sqrt{400}$$

18. The number X of vehicles entering a large service plaza averages 300 per hour. There are many vehicles passing and each has a small probability of entering the plaza. These appear to be independent events so the Poisson model is deemed appropriate for random variable X . Determine a 68% interval for X (answer rounded).

- a. [283, 317] b. [280, 320] c. [160, 280] d. [266, 334] e. [260, 340]

$$300 \pm \sqrt{300}$$

19-21. We are given the probability distribution and summary totals shown:

x	$p(x)$	$x p(x)$	$x^2 p(x)$
-10	0.1	-1	10
0	0.7	0	0
20	0.2	4	80
totals	1.0	3	90

$$300 \pm 17$$

19. $E X^2 =$

- a. 9 b. 500 c. 90 d. 87 e. 81

20. $\text{Var } X =$ (you may use a calculator or employ summary information above)

- a. 9 b. 500 c. 90 d. 87 e. 81

$$E X^2 - (E X)^2 = 90 - 9 = 81$$

21. $P(X^2 > 0) =$

- a. 0.5 b. 1 c. 0.2 d. 0.9 e. 0.3

$$P(X^2 > 0) = 1 - P(X = 0) = 1 - 0.7 = 0.3$$