

1. People are equal-probability selected from a population and sorted by sex and gene type. Which one of the tables below exhibits independence between sex and gene type?

a.		AA	Aa	aa
	male	60	30	20
	female	90	45	30
b.		AA	Aa	aa
	male	20	30	30
	female	30	45	15
c.		AA	Aa	aa
	male	30	30	10
	female	20	45	15
d.		AA	Aa	aa
	male	20	15	10
	female	50	45	20
e.		AA	Aa	aa
	male	40	20	20
	female	60	90	30

60:90 as 30:45 as 20:30

proportionality is the  
APPEARANCE OF INDEPENDENCE

IN TABLED COUNTS

2-4. A plane has two engines. The probability of the left engine failing is 0.002, same as for the right engine failing. Engine failures are independent events.

2. What is the probability that (the left engine fails)  $\cap$  and (the right does not)?

- a.  $0.002 + 0.002 - 0.002 \cdot 0.002$     b.  $0.002 \cdot 0.002$     c.  $0.998 \cdot 0.002$     L Fails R NOT  $0.002 \times 0.998$   
d.  $0.998 \cdot 0.998$     e.  $0.994$

3. What is the probability that (the left engine fails)  $\cup$  or (the right engine fails)?

- a.  $0.002 + 0.002 - 0.002 \cdot 0.002$     b.  $0.002 \cdot 0.002$     c.  $0.998 \cdot 0.002$   
d.  $0.998 \cdot 0.998$     e.  $0.994$     General ADDITION RULE

4. What is the probability that neither engine fails?

- a.  $0.002 + 0.002 - 0.002 \cdot 0.002$     b.  $0.002 \cdot 0.002$     c.  $0.998 \cdot 0.002$   
d.  $0.998 \cdot 0.998$     e.  $0.994$

5-6. A person will be selected at random from all those sorted by sex and gene type in the table below.

	AA	Aa	aa	
male	4	8	5	17
female	6	9	3	18
	10	17	8	35

5. What is the probability the person does not have gene type aa?

- a.  $\frac{6}{18}$  b.  $\frac{27}{35}$  c.  $\frac{4}{10}$  d.  $\frac{20}{35}$  e.  $\frac{13}{35}$

$$\frac{10+17}{35} = \frac{27}{35}$$

6. What is the conditional probability  $P(AA | \text{if female})$ , that the selected person has gene type AA if they are female?

- a.  $\frac{6}{18}$  b.  $\frac{27}{35}$  c.  $\frac{4}{10}$  d.  $\frac{20}{35}$  e.  $\frac{13}{35}$

$$\frac{6}{9+3+18} = \frac{6}{18}$$

$$\frac{6}{18}$$

7-10. Suppose that  $P(OIL) = 0.4$ ,  $P(+ | \text{if } OIL) = 0.7$ ,  $P(+ | \text{if } OIL^c) = 0.2$ .

8.  $P(+)$  =  $P(OIL+) + P(OIL^c+) = P(+ | \text{if } OIL) \times P(OIL) + P(+ | \text{if } OIL^c) \times P(OIL^c)$

- a. 0.6 b. 0.3 c. 0.8 d. 0.7 e. 0.4 =  $0.7 \times 0.4 + 0.2 \times (1-0.4) = 0.28 + 0.12 = 0.4$

7.  $P(- | \text{if } OIL) = 1 - P(+ | \text{if } OIL) = 1 - 0.7 = 0.3$

- a. 0.6 b. 0.3 c. 0.8 d. 0.7 e. 0.4

$$\frac{P(OIL+)}{P(+)} = \frac{0.7 \times 0.4}{0.4} = 0.7$$

9.  $P(OIL | \text{if } +) = \frac{P(OIL+)}{P(+)}$   
 a. 0.6 b. 0.3 c. 0.8 d. 0.7 e. 0.4

Suppose also that the cost to test is 20, the cost to drill is 80, and the gross return from finding OIL is 700.

10. NET return from policy "test, but only drill if the test is positive" for the contingency OIL+ is equal to:

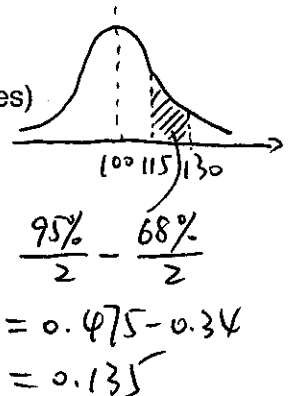
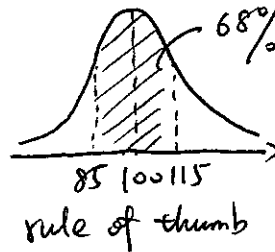
- a. -100 b. 600 c. 580 d. -20 e. 680

Test	Drill	OIL	Return	
-20	-80	+700	=	600

11-12. The distribution of IQ is normal with mean 100 and standard dev 15.  
 $E(X) = 100$        $SD(X) = 15$

11. The probability of IQ in range [100+15, 100+30] is approximately:  
 a. 0.34 b. 0.475 c. 0.135 d. 0.68 e. 0.95 (draw a picture and work with pieces)

12. The probability of an IQ in range [100-15, 100+15] is approximately:  
 a. 0.34 b. 0.475 c. 0.11 d. 0.68 e. 0.95



13-17. A lottery has return  $X$  which is a random variable with

$$E X = 3 \quad \text{Var } X = 4$$

13. Variance of  $(2X + 7) = 2^2 \text{Var } X = 4 \times 4 = 16$

a. 1 b. 8 c. 16 d. 53 e. 23

14. Standard deviation of  $X = \sqrt{\text{Var } X} = \sqrt{4} = 2$

a. 4 b. 1 c. 7 d. 2 e. 3

For 100 independent plays of the lottery define  $T = X_1 + \dots + X_{100}$ .

15.  $E T = 100 E X = 300$

a. 30 b. 7 c. 70 d. 300 e. 3

16.  $\text{Var } T = 100 \text{Var } X = 400$

a. 400 b. 200 c. 20 d. 10 e. 530

17. Approximate 95% interval for  $T = [300 - 2 \times 20, 300 + 2 \times 20] = [260, 340]$   $SD T = \sqrt{\text{Var } T} = \sqrt{400} = 20$

a. [220, 380] b. [280, 320] c. [160, 280] d. [235, 365] e. [260, 340]

18. The number  $X$  of vehicles entering a large service plaza averages 300 per hour. There are many vehicles passing and each has a small probability of entering the plaza. These appear to be independent events so the Poisson model is deemed appropriate for random variable  $X$ . Determine a 95% interval for  $X$  (answer rounded).

a. [283, 317] b. [280, 320] c. [160, 280] d. [266, 334] e. [260, 340]

$$\mu = \lambda = 300$$

$$E X = \lambda = 300$$

19-21. We are given the probability distribution and summary totals shown:  $SD X = \sqrt{\text{Var } X} = \sqrt{300}$

$x$	$p(x)$	$x p(x)$	$x^2 p(x)$
-10	0.1	-1	10
0	0.7	0	0
20	0.2	4	80
totals		1.0	

$$[300 - 2 \times \sqrt{300}, 300 + 2 \times \sqrt{300}] = [266, 334]$$

19.  $P(X^2 > 0) = P(X \neq 0) = P(X = -10) + P(X = 20)$

a. 0.5 b. 1 c. 0.2 d. 0.9 e. 0.3

$$= 0.1 + 0.2 = 0.3$$

$$\begin{matrix} \textcircled{3} \\ E X \\ \textcircled{90} \\ E X^2 \end{matrix}$$

20.  $E X^2 = \sum x^2 p(x) = 90$

a. 9 b. 500 c. 90 d. 87 e. 81

21.  $\text{Var } X =$  (you may use a calculator or employ summary information above)

a. 9 b. 500 c. 90 d. 87 e. 81

$$\text{Var } X = E X^2 - (E X)^2 = 90 - 3^2 = 81$$