KEY to Quiz 3-1-10 prototype 20 minutes
Your quiz may differ. Use this key to help you solve your version.

1. Determine z with P(Z > z) = 0.0116. **Same as P(Z < z) = 1 - 0.0116 = 0.9884.**

0.07

search body of z-table **2.2** $\boxed{0.9884}$ $\Longrightarrow$ $\boxed{z = 2.27}$

2. Determine z with P(|Z| > z) = 0.008. **Same as P(Z > z) = 0.008/2 = 0.004.**

0.0$\mathbf{5}$

search body of z-table **2.$\mathbf{6}$** $\boxed{0.996}$ $\Longrightarrow$ $\boxed{z = 2.65}$

3. A sample of 400 soda cans has been stored for six years on a cold floor. Of these 49 have leaked. Determine the **right end value** of a 68% confidence interval for p = the population fraction of cans that would leak in a similar test. Choose closest value.

68% CI for unknown p is $\boxed{\hat{p} \pm 1\sqrt{\hat{p}\,\hat{q} \,/\, n}}$

= $\boxed{49/400 \pm 1\sqrt{49/400\ 351/400\ /\ 400}}$

= $\boxed{0.1225 \pm 0.0163931}$

whose right end value is $\boxed{0.1225 + 0.0163931}$ **=** $\boxed{0.138893}$**.**

4. Re-do #3 for the Agresti-Coull method, preferred when $\hat{p}$ is near zero. **A-C method adds two "successes" and two "failures to make n+4 trials.**

68% A-C CI for unknown p is $\boxed{\tilde{p} \pm 1\sqrt{\tilde{p}\,\tilde{q} \,/\, n}}$

= $\boxed{51/404 \pm 1\sqrt{51/404\ 353/404\ /\ 404}}$

= $\boxed{0.126238 \pm 0.0165234}$

whose right end value is $\boxed{0.126238 + 0.0165234}$ **=** $\boxed{0.142761}$**.**

5. A sample of 400 soda cans has been stored for six years on a cold floor. Of these 50 have leaked. Determine the P-value for a test of H_0: p = 0.1 versus H_1: p > 0.1. **The sample estimate of p is $\hat{p}$ = 50/400 = 0.125.**

$$\text{test statistic} = \boxed{\frac{\hat{p} - p_0}{\sqrt{p_0\, q_0 / n}}} = \boxed{\frac{\hat{p} - p_0}{\sqrt{p_0\, q_0 / n}}} = \boxed{\frac{0.125 - 0.1}{\sqrt{0.1\ 0.9 / 400}}}$$

which works out to $\boxed{1.67}$. The P-value is the probability of data more against the null hypothesis than ours is. P(Z > 1.67) = 1 - P(Z < 1.67).

0.07

search body of z-table 1.6 $\boxed{0.9525}$

so the P-value is 1-0.9525 = 0.0475. This would occur by chance alone 4.75% of the time even if p = 0.1.

6. If we reject the null hypothesis whenever the P-value is below 0.01 what is the probability we will reject the null hypothesis when the null hypothesis is true?

ans. 0.01.

7. Each of 900 researchers independently prepares a 95% CI for their respective probability of success p. Around how many CI fail to cover their respective p?

ans. 5% of 900 or 45.

8. It is desired to set up a test to distinguish between null hypothesis p = 0.2 and alternative hypothesis p = 0.3 (David vs Goliath setup) with type 1 error probability 0.01 and type 2 error probability 0.02. Determine the required sample size n.

p_0 = 0.2 with type 1 error probability 0.01.
p_1 = 0.3 with type 2 error probability 0.02.

Using the methods of #1 and #2,

P(Z > z_0) = 0.01 gives z_0 = 2.33.

P(Z < z_1) = 0.02 gives z_1 = - 2.05.

The sample size n is determined from

$$\mathbf{n} = \left(\frac{\sqrt{p_0\, q_0}\ \left| z_0 \right| + \sqrt{p_1\, q_1}\ \left| z_1 \right|}{p_0 - p_1} \right)^2$$

$$= \left(\frac{\sqrt{.2\ .8}\ \left|2.33\right| + \sqrt{.3\ .7}\ \left|-2.05\right|}{.2 - .3} \right)^2$$

= 350.224 which we round up to n = 351.

9. Determine c for the test of #8.

$$\mathbf{c} = z_0 \sqrt{n\, p_0\, q_0} + 0.5 + \mathbf{n}\, p_0$$

$$= 2.33 \sqrt{350.224\ 0.2\ 0.8} + 0.5 + 350.224\ \ 0.2$$

$$= 87.9865$$

So the specified test will sample n = 351 and will reject the null hypothosis p = 0.2 in favor of the alternative p = 0.3 if the number of "successes" is greater than or equal to 88.

On your variant of the quiz I asked you to solve for "c" using an "incorrect" n so that you could get #9 right if you knew how to do it but had mistakenly gotten the wrong n in #8.