KEY to Quiz 3-1-10 prototype 20 minutes Your quiz may differ. Use this key to help you solve your version.

2.

1. Determine z with P(Z > z) = 0.0116. Same as P(Z < z) = 1 - 0.0116 = 0.9884.

		0.07		
search body of z-table	2.2	0.9884	\Rightarrow	<i>z</i> = 2.27
Determine z with P($ Z > z$) =	0.008.	Same as P(Z	Z > z) =	0.008/2 = 0.004.
		0.05		
search body of z-table	2. 6	0.996	\Rightarrow	z = 2.65

3. A sample of 400 soda cans has been stored for six years on a cold floor. Of these 49 have leaked. Determine the **right end value** of a 68% confidence interval for p = the population fraction of cans that would leak in a similar test. Choose closest value.

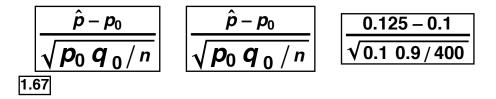
68% CI for unknown p is $\hat{p} \pm 1 \sqrt{\hat{p} \hat{q} / n}$ = $49/400 \pm 1 \sqrt{49/400 \ 351/400 \ / 400}$ = 0.1225 ± 0.0163931 whose right end value is 0.1225 + 0.0163931 = 0.138893.

4. Re-do #3 for the Agresti-Coull method, preferred when \hat{p} is near zero. A-C method adds two "successes" and two "failures to make n+4 trials.

68% A-C CI for unknown p is $\tilde{p} \pm 1 \sqrt{\tilde{p} \tilde{q} / n}$ = $51/404 \pm 1 \sqrt{51/404} 353/404/404$ = 0.126238 ± 0.0165234

whose right end value is 0.126238 + 0.0165234 = 0.142761.

5. A sample of 400 soda cans has been stored for six years on a cold floor. Of these 50 have leaked. Determine the P-value for a test of H₀: p = 0.1 versus H₁: p > 0.1. The sample estimate of p is $\hat{p} = 50/400 = 0.125$.



0.9525

H₀

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p

test statistic =
$$\frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} = \frac{0.125 - 0.1}{\sqrt{0.1 0.9 / 400}}$$

which works out to 1.67. The P-value is the probability of data more against the null hypothesis than ours is. P(Z > 1.67) = 1 - P(Z < 1.67).

0.07

search body of z-table 1.6 0.9525

so the P-value is 1-0.9525 = 0.0475. This would occur by chance alone 4.75% of the time even if p = 0.1.

6. If we reject the null hypothesis whenever the P-value is below 0.01 what is the probability we will reject the null hypothesis when the null hypothesis is true?

ans. 0.01.

7. Each of 900 researchers independently prepares a 95% CI for their respective probability of success p. Around how many CI fail to cover their respective p?

ans. 5% of 900 or 45.

8. It is desired to set up a test to distinguish between null hypothesis p = 0.2 and alternative hypothesis p = 0.3 (David vs Goliath setup) with type 1 error probability 0.01 and type 2 error probability 0.02. Determine the required sample size n.

 $p_0 = 0.2$ with type 1 error probability 0.01. $p_1 = 0.3$ with type 2 error probability 0.02.

Using the methods of #1 and #2,

$$P(Z > z_0) = 0.01$$
 gives $z_0 = 2.33$.

 $P(Z \le z_1) = 0.02$ gives $z_1 = -2.05$.

The sample size n is determined from

$$n = \left(\frac{\sqrt{p_0 q_0} |z_0| + \sqrt{p_1 q_1} |z_1|}{p_0 - p_1}\right)^2 \left(\frac{\sqrt{.2 .8} |2.33| + \sqrt{.3 .7} |-2.05|}{.2 - .3}\right)^2$$

$$z_0 \sqrt{n p_0 q_0} p_0$$

 $\sqrt{350.224 \ 0.2 \ 0.8}$

H₁

$$= \left(\frac{\sqrt{.2.8} |2.33| + \sqrt{.3.7} |-2.05|}{.2 - .3}\right)^2$$

= 350.224 which we round up to n = 351.

9. Determine c for the test of #8.

$$c = z_0 \sqrt{n p_0 q_0} + 0.5 + n p_0$$

= 2.33 \sqrt{350.224} \cdot 0.2 \cdot 0.8 + 0.5 + 350.224 \cdot 0.2
= 87.9865

So the specified test will sample n = 351 and will reject the null hypothosis p = 0.2 in favor of the alternative p = 0.3 if the number of "successes" is greater than or equal to 88.

On your variant of the quiz I asked you to solve for "c" using an "incorrect" n so that you could get #9 right if you knew how to do it but had mistakenly gotten the wrong n in #8.