Answer Key

z-table use

1. Determine table entries for look up of $P(Z < -2.61)$.

a. row -2.61 column 0.00 b. row 2.61 column 0.00

c. row 2.6 column 0.01 d. row -2.6 column 0.01

2. Determine $P(Z < -2.61)$.

a. 0.0045 b. 0.1711 c. 0.0289 d. 0.0023 e. 0.0087

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3. Closest table entry found when looking up of z satisfying $P(Z < z) = 0.864$.

a. 0.8631 b. 0.8599 c. 0.8616 d. 0.8461 e. 0.8643

4. Value z satisfying $P(Z < z) = 0.864$ (use closest table entry).

a. 1.02 b. 1.10 c. 2.13 d. 0.94 e. 0.90

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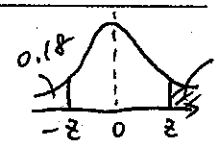
19 4. The z satisfying $P(Z > z) = 0.18$ also satisfies $P(Z < -z) = ?$



0.18 symmetric property

a. 0.18 b. -0.18 c. 0.09 d. 0.13 d. 0.22

5. Determine z satisfying $P(Z > z) = 0.18$ (use closest table entry). $P(Z < z) = 1 - 0.18 = 0.82$

a. 0.77 b. 0.92 c. 0.84 d. 1.13 e. 1.01

Use z-table to get z as 0.92

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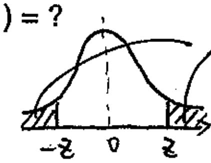
6. The z satisfying $P(|Z| > z) = 0.20$ also satisfies $P(Z > z) = ?$

$= P(Z > z \text{ or } Z < -z)$



By symmetric property,

$P(Z > z) = \frac{0.20}{2} = 0.10$

a. 0.10 b. 0.90 c. 0.15 d. 0.20 e. 0.30

CI for p in Bernoulli trials

7. What fraction of **80%** CI for p cover their intended target p?

a. ~1.0 b. around 0.90 c. exactly 0.90 d. exactly 0.8 e. around 0.8

$n=40$ $N=310$

8. A **with**-replacement sample of 40 hospital admittees from a population of 310 admittees finds 13 lack insurance for the recommended treatment. The CI for the corresponding population fraction p is:

$X=13$

a. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}$ b. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}\sqrt{\frac{N-n}{N-1}}$ c. $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}$

d. $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}\sqrt{\frac{N-n}{N-1}}$ e. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}$

$\hat{p} = \frac{X}{n} = \frac{13}{40} = 0.325$

9. The **95%** CI for p (**not** 90% CI as in #7) is from #8 $\hat{p} \pm z\cdot\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.325 \pm 1.96 \times \sqrt{\frac{0.325 \times 0.675}{40}}$

a. [0.200847, 0.480972] b. [0.189646, 0.460354] c. [0.110236, 0.539764] $= 0.325 \pm 0.145151$
d. [0.236132, 0.413868] e. [0.179849, 0.470151]

$\Rightarrow [0.179849, 0.470151]$

10. The Agresti-Coull estimate of p, different from $\hat{p}$ is $\tilde{p}$ equal to $\tilde{p} = \frac{X+2}{n+4} = \frac{15}{44} \doteq 0.340909$

a. 0.340909 b. 0.220395 c. 0.199699 d. 0.340909 e. 0.254910

Remark: a and d are the same

11. The Agresti-Coull 95% CI for p is $\tilde{p} \pm z\sqrt{\frac{\tilde{p}\tilde{q}}{n+4}} = 0.340909 \pm 1.96 \times \sqrt{\frac{0.340909 \times 0.659091}{44}}$

a. [0.200847, 0.480972] b. [0.302811, 0.638366] c. [0.110236, 0.539764] $= 0.340909 \pm 0.140062$
d. [0.236132, 0.413868] e. [0.179849, 0.470151]

$\Rightarrow [0.200847, 0.480971]$

$n=40$ $N=300$

12. A **without**-replacement sample of 40 hospital admittees from a population of 300 admittees finds 13 lack insurance for the recommended treatment. The form of a **95%** CI for the corresponding population fraction p is:

$X=13$

a. $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}$ b. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}$ c. $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}\sqrt{\frac{N-n}{N-1}}$

d. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}\sqrt{\frac{N-n}{N-1}}$ e. $\hat{p} \pm z\sqrt{n\hat{p}\hat{q}}$

$\hat{p} = \frac{\;}{n} = \frac{\;}{40} = 0.325$

$\hat{p} \pm z \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} \cdot \sqrt{\frac{N-n}{N-1}} = 0.325 \pm 1.96 \times \sqrt{\frac{0.325 \times 0.675}{40}} \times \sqrt{\frac{300-40}{300-1}} = 0.325 \pm 0.135354$

$\Rightarrow [0.189646, 0.460354]$

13. The above **95%** CI for p is

a. [0.200847, 0.480972] b. [0.189646, 0.460354] c. [0.110236, 0.539764]
d. [0.236132, 0.413868] e. [0.189646, 0.460354]

Remark: b and e are the same.

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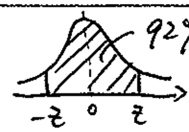
14. For a 92% CI what z would be used?

a. 1.45 b. 1.65 c. 1.34 d. 1.25 e. 1.75



$P(Z < z) = 0.92 + \frac{1-0.92}{2} = 0.96$

Use z-table to get z as 1.75.

tests of hypotheses

15. It is desired to test the null hypothesis p = 0.2 versus the alternative p > 0.2. where p is the fraction of snow blower sales having a service contract in the deal. The rate 0.2 applied just prior to a new advertising "rollout" promoting the service contract. Give the form of the z-test statistic. Note the following: This is not a CI. The test is not being set up in terms of X.

a. $\dfrac{\hat{p} - p_0}{\sqrt{\hat{p}\hat{q}/n}}$ b. $\dfrac{\hat{p} - p_0}{\sqrt{n\hat{p}\hat{q}}}$ c. $\hat{p}$ - p_0 d. $\dfrac{\hat{p} - p_0}{\sqrt{n p_0 q_0}}$ e. $\dfrac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$

16. A sample of 100 ($n=100$) snow blower sales finds 27 ($X=27$) had the contract as part of the deal. Determine the P-value for this data employing your choice in #15.

$p\text{-value} = P\left(z > \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}\right) = P\left(z > \frac{27/100 - 0.2}{\sqrt{\frac{0.2 \times 0.8}{100}}}\right)$

$= P(z > 1.75) = 0.0401$

a. 0.0401 b. 0.0211 c. 0.03893 d. 0.10560 e. 0.09284

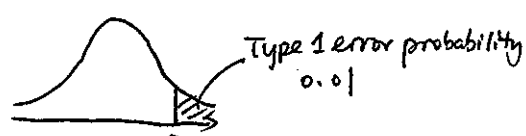
tests with pre-assigned probabilities of type 1 and type 2 error

17. It is desired to test

H_0: p = 0.2 type one error probability 0.01
H_A: p = 0.4 type two error probability 0.04

Determine Z_0.

$P(Z_1 > z_0) = 0.01 \Rightarrow P(Z_1 < z_0) = 0.99$



Use z-table to get z_0 as 2.33

a. 2.59 b. 1.75 c. 2.33 d. 2.67 e. 2.81

18. You are given that Z_1= -1.75. Using the information in #17 determine the sample size n required to achieve the goals set out above.

a. 231 b. 81 c. 135 d. 306 e. 78

$n = \left(\frac{\sqrt{0.2 \times 0.8} \times |2.33| + \sqrt{0.4 \times 0.6} \times |-1.75|}{0.2 - 0.4} \right)^2$

$= 80.0418 \uparrow$ round it up to $\underline{\underline{81}}$.

$\sqrt{(N-n)/(N-1)}$

$\sqrt{npq}$

$\sqrt{pq/n}$

$\sqrt{\hat{p}\hat{q}/n}$

$$\left(\frac{\sqrt{p_0 q_0}\,|z_0| + \sqrt{p_1 q_1}\,|z_1|}{p_0 - p_1} \right)^2$$

$z_0 \sqrt{n p_0 q_0} + 0.5 + n p_0$